Hide and Seek among Procrastinators

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Abstract

Two sophisticated, present-biased agents must each complete a task by a deadline, and completing on the same date costs less. An agent may complete early to avoid a partner whose company would tempt her to delay. For any horizon, avoidance requires at least three periods and is impossible under full naiveté. If the partner seeks her company, they play hide-and-seek. With three periods this happens on an open set of parameters, where completion dates are random, company can hurt both agents, and an agent's cost can rise with her partner's self-control. Observable, irreversible starts lead to joint completion.

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1 Introduction

Many unpleasant tasks are easier to do in company. Peer effects have been documented in vaccination decisions (Rao, Mobius and Rosenblat, 2007), exercise (Carrell, Hoekstra and West, 2011; Babcock et al., 2015) and workplace productivity (Mas and Moretti, 2009; Bandiera, Barankay and Rasul, 2010), and evidence from the workplace suggests that peers also affect self-control (Kaur, Kremer and Mullainathan, 2010). When the people involved are present-biased, the option to work together turns individual timing decisions into a dynamic game. A moderate procrastinator, who on her own would complete the task one date before the deadline, may then finish *earlier* when paired with a severe procrastinator, who on her own would wait until the deadline, because she expects that her partner's company at the deadline would tempt her into further delay. Cerrone (2021), who introduced this game under the name *procrastination game*, calls this effect the *avoidance of bad company*.

This paper studies what happens when such an agent is paired with a partner who would follow her. An agent who fears her partner's future company prefers to complete the task on a date on which the partner does not. If the partner prefers to complete on the same date as she does, the two play a game of *hide-and-seek*. That game has no equilibrium in pure strategies, so completion dates are random. In the three-period model the pair then completes together at the deadline with positive probability, which is the outcome the first agent was trying to avoid.

Consider Ann and Bob, who must each write an essay by Wednesday. Writing costs 1 on Monday, 2 on Tuesday and 10 on Wednesday, and writing on the same day as one's friend lowers the cost by a quarter. Both are sophisticated quasi-hyperbolic discounters, with present-bias factors of 0.12 for Ann and 0.25 for Bob. Alone, Bob writes on Tuesday. Ann knows that she would put the essay off until Wednesday if she did not write on Monday, so she writes on Monday. When they can write together, each one's choice on Monday depends on the other's. If Bob writes on Monday, Ann joins him, because company makes Monday cheap. If Bob waits, so does Ann, since she would rather write on Wednesday with Bob than on Monday alone. Bob's incentives are different. If Ann writes on Monday, he waits, because company on Monday is not worth giving up the option of writing on Tuesday, when no one will be around to tempt him. If Ann waits, however, Bob writes on Monday alone. He knows that on Tuesday, with Ann planning to write on Wednesday, he would wait for her, and he would rather pay 1 now than end up writing on Wednesday. Table 1 shows the resulting game on Monday.

Table 1: The Monday game between Ann and Bob

	Bob writes (Y)	Bob waits (N)
Ann writes (Y)	0.75, **0.75**	1, **0.5**
Ann waits (N)	1.2, **1**	0.9, **1.875**

Notes: Entries are the costs perceived by Monday's selves (Ann first, Bob in bold), given equilibrium play on Tuesday and Wednesday. Costs are $(c_1, c_2, c_3) = (1, 2, 10)$, joint completion costs $0.75\,c_t$, and $(\beta_{\text{Ann}}, \beta_{\text{Bob}}) = (0.12, 0.25)$. Ann's best response matches Bob's action and Bob's best response mismatches Ann's, so best responses cycle and no pure-strategy equilibrium exists.

The Monday game has no equilibrium in pure strategies, and the unique equilibrium of the full game is in mixed strategies. Ann writes on Monday with probability $7/9$ and Bob with probability $2/11$, and with probability $2/11$ both end up writing on Wednesday. Ann's expected cost rises from 1 to 2.51 and Bob's from 2 to 2.78, so both are worse off than if they worked alone, although company lowers the cost of every essay written together. Suppose instead that each could see whether the other had started, that starting committed the writer to finishing that day, and that an essay written later the same day still earned the discount. Then Bob would start on Monday morning, Ann would write that day too, and each would pay 0.75.

I study the procrastination game for an arbitrary horizon. Along the path on which neither agent has completed the task, the game is a chain of 2×2 stage games. In each of them, a player's incentives are summarized by whether she would complete today if her partner does and whether she would complete today if her partner does not. The answers classify each player at each date as *eager*, *reluctant*, a *seeker* (she completes today if and only if her partner does) or an *avoider* (she completes today if and only if her partner does not). Two seekers play a coordination game, two avoiders play chicken, and a seeker and an avoider play hide-and-seek.

Section 3 gives necessary conditions for avoidance (Theorem 1). Avoidance requires present bias, since a time-consistent agent is always eager, and it is impossible under full naiveté, because a fully naive agent does not anticipate her own future procrastination (Proposition 8). A sophisticated avoider must also expect that, if both wait, her partner's presence will make her complete later than she would on her own, with positive probability, and that the resulting increase in cost will exceed the value of company. Avoidance is also impossible at the last decision date, so hide-and-seek requires at least three periods. The same section characterizes the dates on which the two agents can complete together (Proposition 3). Joint completion on the first date is sustained by each agent's fear of being

left alone, which can be stronger for more present-biased agents because sophisticated behavior is not monotone in present bias.

For the three-period game, Section 4 characterizes equilibrium for pairs of a "late" agent, who does not complete on the penultimate date even if her partner does, and a "moderate" agent, who would complete on the penultimate date alone but would rather wait for company at the deadline (Theorem 2). This class includes every pair of a severe and a moderate procrastinator in which the avoidance of bad company can arise. Hide-and-seek occurs if and only if the moderate agent prefers completing on the penultimate date alone to completing on the first date in company and the late agent is willing to complete early in company but not alone. Such pairs exist if and only if $c_2^2 < \kappa c_1 c_3$, where c_t is the cost on date t and κ the cost factor under company, and they are the only pairs for which the three-period game has no equilibrium with a deterministic outcome. The condition holds when costs rise slowly at first and steeply near the deadline.

Section 5 studies welfare. In the unique equilibrium, company can make both partners strictly worse off than working alone, even though neither is a severe procrastinator (Theorem 3). The value of a partner is also not monotone in the partner's self-control (Proposition 6). A moderate procrastinator gains when a late partner becomes willing to join her early, but her cost then rises with the partner's self-control. When $c_1 < \kappa^2 c_2$, as in the example above, her cost rises above what she would pay working alone and falls back to that level once the partner is willing to complete early alone.

Section 6 studies observable starts. Suppose that a player can see her partner start and join her later the same day, that starting commits a player to finishing that day, and that joining later preserves the full value of company. Then, at every date at which the agents would otherwise play hide-and-seek, the avoider starts in the morning and both complete that day (Proposition 7). Starting in the morning gives the avoider company that day and leaves no partner behind to tempt her later. The seeker follows, although she may also start in the morning. In the three-period game, observability never hurts either agent when the simultaneous-move equilibrium is unique, and it gives both agents the best feasible outcome when that equilibrium is in mixed strategies (Theorem 4). With longer horizons, however, observability can hurt an agent, because the prospect of hide-and-seek at a later date can make her earlier selves complete early (Example 1).

Related literature. The paper belongs to the literature on present bias that begins with Strotz (1955), Phelps and Pollak (1968) and Laibson (1997), and in particular to the work of O'Donoghue and Rabin (1999*a*,*b*, 2001, 2008) on procrastination. Ericson and Laibson (2019)

review this literature, and Ariely and Wertenbroch (2002), DellaVigna and Malmendier (2006), Augenblick, Niederle and Sprenger (2015), Kaur, Kremer and Mullainathan (2015) and Bisin and Hyndman (2020) provide evidence. An isolated sophisticate in the present model behaves as in O'Donoghue and Rabin (1999*a*). Her completion date is not monotone in her present bias or in the cost schedule (Section 2), and this non-monotonicity underlies both avoidance and the fear of being left alone.

The closest paper is Cerrone (2021), which introduced the procrastination game and characterized its pure-strategy equilibria with three dates. She classified agents into the four types used in Section 4, noted that the avoidance of bad company requires present bias and sophistication, and showed that two agents of the same type can always behave as they would alone and that two non-procrastinators may coordinate on a later date. She also studied matching, longer horizons and the question of which player would want to move first. Theorem 1(a), Proposition 8 and Corollary 1(a)–(b) extend her results on avoidance and coordination to an arbitrary horizon, and Proposition 7 answers the question about the order of moves. Her analysis also predicts that, under conditions that hold in part of the hide-and-seek region, a severe and a moderate procrastinator both complete on the first date. Corollary 2 shows that no pure-strategy equilibrium exists there. Relative to her paper, I allow an arbitrary horizon and mixed strategies, and I study welfare, including the value of company, and observable starts.

Other papers study present-biased agents who interact. Brocas and Carrillo (2001) and Sogo (2019) show how competition and cooperation on joint or rival tasks affect procrastination. In Battaglini, Bénabou and Tirole (2005), peers convey information about one's own self-control, and in Fahn and Hakenes (2019), teamwork sustained by relational contracts disciplines present-biased workers. Gans and Landry (2016) study teams with naive members, and Hattori (2026*a*,*b*) study procrastination traps and endogenous leadership in teams. In Hattori (2026*b*), guilt from breaking joint commitments generates collective procrastination that "self-starters" can break. Leadership in the present paper instead comes from anticipated temptation by a partner's future company and requires neither guilt nor a prior joint plan. Weinschenk (2016) studies procrastination caused by free-riding in teams of time-consistent agents. Games among time-inconsistent players are also studied by Sarafidis (2006), Akin (2007), Chade, Prokopovych and Smith (2008), Lu (2016), Schweighofer-Kodritsch (2018) and Flesch et al. (2026).

The avoider's motive is related to the demand for commitment of a sophisticated agent who anticipates temptation (Gul and Pesendorfer, 2001; Bénabou and Tirole, 2004) and to the value such an agent places on not acquiring information that would weaken her

resolve (Carrillo and Mariotti, 2000; Bénabou and Tirole, 2002). In the present model the agent has no commitment device, and completing early is the only way she can avoid her partner's future company. The results on observable starts relate to work on endogenous timing, which asks when players prefer to move first or second (Hamilton and Slutsky, 1990; van Damme and Hurkens, 1999), and on leading by example, in which a leader's observable effort induces others to follow (Hermalin, 1998). The predictions that peer effects in procrastination can be non-monotone in peers' types and can depend on whether starts are observable are relevant for the empirical study of social interactions (Manski, 2000).

Outline. Section 2 presents the model. Sections 3 to 6 contain the results described above, Section 7 discusses extensions, and Section 8 concludes. The proof of Theorem 2 is in the main text; the remaining proofs are in Appendix A. Appendix B explains how the equilibria of the three-period game are computed, and Appendix C describes the replication files.

2 Model

2.1 Environment

Two players, $i \in \{1,2\}$, must each complete a task. Throughout, j denotes the player other than i. There are $T \geq 2$ dates, or periods. Each player completes her task exactly once, at a date $\tau_i \in \{1,\dots,T\}$ of her choosing. The task yields a benefit after date T that does not depend on τ_i, which I normalize to zero. Completing the task at date t costs c_t, where $0 < c_1 < c_2 < \cdots < c_T$, unless the partner completes her own task on the same date, in which case it costs κc_t, with $\kappa \in (0,1)$. The tasks are individual. Neither task requires the other player's input, and company is the only link between the players. Player i's realized cost is therefore

$$x_i = \begin{cases} \kappa c_{\tau_i} & \text{if } \tau_i = \tau_j, \\ c_{\tau_i} & \text{if } \tau_i \neq \tau_j. \end{cases}$$

Assumption 1 (Company never justifies delay)**.** $\kappa c_{t+1} > c_t$ for all $t \in \{1,\dots,T-1\}$.

Assumption 1 states that completing on date $t+1$ in company is more costly than completing on date t alone. Hence every postponement raises the long-run cost, so that any delay in the model is a delay that the agent's long-run self would not choose. Following O'Donoghue and Rabin (1999*a*), I call such delay procrastination. The assumption requires

costs to rise by more than the factor $1/\kappa$ from each date to the next, so that $c_T > \kappa^{-(T-1)} c_1$. The dates of the model should therefore be read as a few discrete opportunities separated by substantial cost increases.

Each player has quasi-hyperbolic preferences (Phelps and Pollak, 1968; Laibson, 1997) with present-bias factor $\beta_i \in (0, 1]$ and long-run discount factor one. The self of player i at date t, who has not yet completed the task, evaluates completion on date $\tau_i \geq t$ at realized cost x_i by

$$U_t^i = \begin{cases} -x_i & \text{if } \tau_i = t, \\ -\beta_i x_i & \text{if } \tau_i > t. \end{cases} \tag{1}$$

Player i is present-biased if $\beta_i < 1$ and time-consistent if $\beta_i = 1$.[1] Both players are sophisticated, in that each correctly anticipates the behavior of her own future selves. The game has complete information, and at the end of every date each player observes whether her partner has completed the task. Section 6 lets players observe each other's starts within a date as well.

2.2 Strategies and equilibrium

Following Strotz (1955) and O'Donoghue and Rabin (1999*a*), I treat each self as a separate decision maker. Because a player who has completed the task takes no further action, and because the date on which the partner completed is payoff-irrelevant once it has passed, only two kinds of histories matter at date t. At the *joint history* h_t, neither player has completed before t. At a *solo history*, player i has not completed but player j has. At date T any player who has not completed must do so. A (behavior) strategy profile σ specifies, for every date $t < T$, every player i and every date-t history at which i has not completed, the probability with which self t of player i completes the task.

Definition 1. A strategy profile is an *equilibrium* if, at every history, the behavior of every self who moves at that history maximizes her expected utility (1), given the strategies of all other selves.

An equilibrium is a subgame-perfect equilibrium of the game among the $2T$ selves, in which selves may randomize. Each equilibrium induces a probability distribution over

[1] With a long-run discount factor $\delta \in (0, 1]$, self t's utility from completing on date $\tau \geq t$ at cost x is $-\beta_i^{\mathbf{1}\{\tau > t\}} \delta^{\tau - t} x$. Multiplying by δ^t shows that this is the model in the text with costs $\delta^\tau c_\tau$. All results apply to these discounted costs whenever they satisfy Assumption 1, with welfare also measured in discounted costs. Because the benefit accrues after T regardless of the completion date, it does not affect any decision.

pairs of completion dates (τ_1, τ_2), which I call its *outcome*. The outcome is *deterministic* if this distribution is degenerate and *random* otherwise.

Several statements below involve strict inequalities between costs and discounted continuation costs. A statement holds *for generic parameters* if it holds for all $(c, \kappa, \beta_1, \beta_2)$ outside a set of Lebesgue measure zero. Lemma A.1 in Appendix A shows that, for generic parameters, no self is ever indifferent between completing and waiting against a pure action of her partner, in any equilibrium. All results are stated for generic parameters. Statements about time-consistent players ($\beta_i = 1$, itself a measure-zero case) hold for all parameters, because the relevant inequalities are then strict under Assumption 1. Statements about identical players ($\beta_1 = \beta_2$, also a measure-zero case) hold for generic (c, κ, β) with $\beta_1 = \beta_2 = \beta$, by the same proof, which perturbs only the costs.

2.3 Benchmark: working alone

At a solo history, player i's partner no longer affects her, and her remaining selves play the sequential game studied by O'Donoghue and Rabin (1999*a*).

Lemma 1 (Working alone)**.** *For generic parameters, at a solo history at date t player i's selves have a unique equilibrium continuation, in which she completes the task on date $\tau_i(t)$, defined recursively by $\tau_i(T) = T$ and, for $s < T$,*

$$\tau_i(s) = \begin{cases} s & \text{if } c_s < \beta_i \, c_{\tau_i(s+1)}, \\ \tau_i(s+1) & \text{otherwise.} \end{cases}$$

Let $S_i(t) := c_{\tau_i(t)}$ denote the realized cost of player i's solo continuation from date t, and call $\tau_i(1)$, the date on which she completes when she works alone throughout, her *solo date*. Note that $S_i(t) \geq c_t$. A sophisticate completes today if and only if today's cost is below the discounted cost of the date on which her future selves would complete. As O'Donoghue and Rabin (1999*a*) emphasize, the resulting completion date is not monotone in β_i. A more present-biased agent may complete earlier because she knows that, if she waited, she would wait for a long time. Both avoidance and the fear of being left alone, discussed below, rely on this non-monotonicity.

2.4 Welfare

Following O'Donoghue and Rabin (1999*a*), I evaluate outcomes from the long-run perspective, which weights all dates equally. Player i's *long-run cost* is her expected realized

cost, $K_i := \mathbb{E}[x_i]$. By Assumption 1, the best feasible outcome for both players is joint completion on date 1, at cost κc_1 each. When it matters, I also report the welfare of current selves.

3 Equilibrium structure

3.1 A chain of stage games

Fix a strategy profile σ. For $t < T$, let $J_i(t+1)$ denote player i's expected realized cost under σ from the joint history h_{t+1} on, whether or not h_{t+1} is reached, and set $J_i(T) = \kappa c_T$. At h_t, self t of player i faces four possible situations. If both players complete, she pays κc_t. If she completes and her partner does not, she pays c_t. If her partner completes and she does not, she is alone from date $t+1$ on and, by Lemma 1, will pay $S_i(t+1)$, which she discounts by β_i. If neither completes, play moves to h_{t+1} and she expects to pay $J_i(t+1)$, again discounted by β_i. Selves t of the two players thus play the stage game G_t, whose costs for player i are

	j completes (Y)	j waits (N)
i completes (Y)	κc_t	c_t
i waits (N)	$\beta_i S_i(t+1)$	$\beta_i J_i(t+1)$

(2)

Lemma 2 (Recursive structure)**.** *For generic parameters, a strategy profile σ is an equilibrium if and only if (i) at every solo history the remaining player follows the solo continuation of Lemma 1, and (ii) at every joint history h_t with $t < T$, the completion probabilities of the two selves form a Nash equilibrium of G_t, where $J(t+1) = (J_1(t+1), J_2(t+1))$ is the continuation cost induced by σ.*

Along the path on which neither player has completed, the game is a chain of 2×2 games linked by continuation costs, and every departure from this path leads to a solo continuation. The chain can be solved backward. The continuation cost $J(T) = (\kappa c_T, \kappa c_T)$ determines G_{T-1}, each Nash equilibrium of G_{T-1} determines $J(T-1)$ and hence G_{T-2}, and so on.

Proposition 1 (Existence)**.** *For generic parameters, every stage game G_t arising in an equilibrium has either exactly one or exactly three Nash equilibria. An equilibrium exists, and there are at most 3^{T-1} equilibria.*

3.2 Motives

The stage game G_t is governed by two comparisons for each player.

Definition 2 (Motives)**.** Fix an equilibrium and a joint history h_t, $t < T$. Player i is *willing to join* at h_t if $\kappa c_t < \beta_i S_i(t+1)$, that is, if she prefers to complete today when her partner does. She is *willing to go alone* at h_t if $c_t < \beta_i J_i(t+1)$, that is, if she prefers to complete today when her partner waits. Player i is

- *eager* if she is willing to join and willing to go alone;
- *reluctant* if she is neither;
- a *seeker* if she is willing to join but not willing to go alone; and
- an *avoider* if she is willing to go alone but not willing to join.

The four motives determine the stage game.

Proposition 2 (Four stage games)**.** *For generic parameters, at every joint history h_t with $t < T$:*

(a) If at least one player is eager or reluctant, G_t has a unique Nash equilibrium, which is in pure strategies.

(b) If both players are seekers, G_t is a coordination game, with Nash equilibria (Y,Y), (N,N) and a completely mixed equilibrium.

(c) If both players are avoiders, G_t is a game of chicken, with Nash equilibria (Y,N), (N,Y) and a completely mixed equilibrium.

(d) If one player is a seeker and the other an avoider, G_t is a game of hide-and-seek, and its unique Nash equilibrium is completely mixed.

In every completely mixed equilibrium of G_t, player j completes with probability

$$\pi_j = \frac{c_t - \beta_i J_i(t+1)}{\left[c_t - \beta_i J_i(t+1)\right] + \left[\beta_i S_i(t+1) - \kappa c_t\right]}. \tag{3}$$

Table 2 summarizes. As in any mixed equilibrium, the probability (3) with which j completes is determined by i's indifference and depends only on i's payoffs. Section 4 uses this to derive comparative statics.

3.3 Necessary conditions for avoidance

Chicken and hide-and-seek require avoiders. Theorem 1 gives necessary conditions for a player to be an avoider.

Table 2: Motives and stage games

Motive of i	If j completes	If j waits	Defining conditions
Eager	Y	Y	$\kappa c_t < \beta_i S_i(t+1),\ c_t < \beta_i J_i(t+1)$
Reluctant	N	N	$\kappa c_t > \beta_i S_i(t+1),\ c_t > \beta_i J_i(t+1)$
Seeker	Y	N	$\kappa c_t < \beta_i S_i(t+1),\ c_t > \beta_i J_i(t+1)$
Avoider	N	Y	$\kappa c_t > \beta_i S_i(t+1),\ c_t < \beta_i J_i(t+1)$
Stage game G_t by pair of motives			
Eager or reluctant, any			Dominance solvable; unique equilibrium, pure
Seeker, seeker			Coordination; (Y,Y), (N,N), and mixed
Avoider, avoider			Chicken; (Y,N), (N,Y), and mixed
Seeker, avoider			Hide-and-seek; unique equilibrium, mixed

Notes: The second and third columns give player i's best response at the joint history h_t. $S_i(t+1)$ is the cost of i's solo continuation and $J_i(t+1)$ her expected cost if both wait.

Theorem 1 (Necessary conditions for avoidance)**.** *Suppose that, in some equilibrium, player i is an avoider at the joint history h_t. Then:*

(a) Present bias. $\beta_i < \kappa c_t / S_i(t+1) < \kappa$. *A time-consistent player is eager at every joint history and completes on date 1 in every equilibrium.*

(b) Anticipated contagion. $J_i(t+1) > S_i(t+1)/\kappa$. *Consequently, conditional on reaching h_{t+1}, with positive probability player i completes strictly later than $\tau_i(t+1)$, the date on which she would complete if her partner were no longer around.*

(c) Horizon. $t \leq T-2$. *In particular, avoiders, chicken and hide-and-seek can arise only if $T \geq 3$.*

An avoider prefers to complete today alone rather than wait together with her partner, but she prefers to wait if her partner completes today. Both preferences can hold only if her partner's continued presence raises her cost. If both wait, the partner's company on later dates leads the avoider's future selves to complete later than they would on their own, and by part (b) the resulting increase in cost must exceed the value of company. A time-consistent agent has no such problem (part (a)). At the last decision date, the only joint continuation is completion together at the deadline, which costs less than completing at the deadline alone, so avoidance is impossible there (part (c)). Proposition 8 in Section 7 shows that avoidance is also impossible under full naiveté, because a naive agent expects her future selves to complete promptly.

The avoidance of bad company described in the introduction is the case in which the avoider's partner is reluctant, so that the avoider completes today and her partner waits

(Proposition 2(a)). If the partner is a seeker, completing today does not separate the two, because the seeker would complete today as well.

3.4 Joint completion

Proposition 3 (Joint completion dates)**.** *For $d \in \{1, \dots, T\}$, there is an equilibrium in which both players complete on date d with probability one if and only if*

(i) $d = T$, *or* $\kappa c_d < \beta_i S_i(d+1)$ *for* $i = 1, 2$; *and*

(ii) $d = 1$, *or* $c_1 > \kappa \beta_i c_d$ *for* $i = 1, 2$.

Condition (i) requires that neither player want to stay behind when the other completes on date d, and condition (ii) that neither want to complete alone before d.

Corollary 1.

(a) Equal solo dates. *If the players' solo dates coincide, $\tau_1(1) = \tau_2(1)$, joint completion on that date is an equilibrium outcome. In particular, this holds for identical players ($\beta_1 = \beta_2$).*

(b) Joint completion at the deadline. *Joint completion on date T is an equilibrium outcome if and only if $\beta_i < c_1/(\kappa c_T)$ for $i = 1, 2$. This includes players who would complete on date 1 alone, namely any β_i with $c_1 < \beta_i c_T < c_1/\kappa$.*

(c) Joint completion on date 1. *Joint completion on date 1 is an equilibrium outcome if and only if $\kappa c_1 < \beta_i S_i(2)$ for $i = 1, 2$. The set of β_i satisfying this condition need not be an interval. With $T = 3$ and $c_2^2 < \kappa c_1 c_3$, it equals $(\kappa c_1/c_3,\ c_2/c_3) \cup (\kappa c_1/c_2,\ 1]$.*

By part (b), company can sustain joint completion at the deadline even among agents who would complete on date 1 alone. If her partner completes on date 1, a player continues alone at cost $S_i(2)$, and by part (c) joint completion on date 1 is an equilibrium outcome only if this prospect is bad enough for both players. I refer to this as the *fear of being left alone*. Because sophisticated behavior is not monotone in present bias, the fear can be stronger for a more present-biased player. With three dates, a player with β_i just below c_2/c_3 would wait until the deadline if left alone, so she is willing to complete on date 1 in company. A moderate procrastinator with $\beta_i \in (c_2/c_3, \kappa c_1/c_2)$ would complete on date 2 if left alone, and she is not willing to complete on date 1 in company. If also $\beta_i < c_2/(\kappa c_3)$, she is an avoider on date 1 whenever her partner is late (Section 4).

3.5 Hopeless partners

Call player j *hopeless* if $\beta_j < \kappa c_1/c_T$. A hopeless player does not complete before the deadline under any circumstances, and her partner's problem reduces to a single-agent problem with a cheaper deadline.

Proposition 4 (Hopeless partners)**.** *Suppose player j is hopeless. Then the equilibrium outcome is unique. Player j completes on date T, and player i completes on date $\hat{\tau}_i$, the date on which a sophisticate with present bias β_i would complete if she faced the cost schedule $\hat{c} = (c_1, \ldots, c_{T-1}, \kappa c_T)$ alone.*

A hopeless partner lowers the cost of completing at the deadline. Because sophisticated behavior is not monotone in the cost schedule, this can delay or advance player i's completion. For every cost schedule satisfying Assumption 1, a player with $c_1 < \beta_i c_T < c_1/\kappa$ completes on date 1 alone but on date T with a hopeless partner. With $T = 3$, a moderate procrastinator ($\tau_i(1) = 2$) with $c_1 < \kappa\beta_i c_3 < c_2$ instead completes on date 1, which is the avoidance of bad company. With longer horizons the acceleration can exceed one date.[2] Since a hopeless partner never follows, completing early always succeeds in avoiding her.

4 Hide-and-seek in the three-period game

This section studies the game with three dates. Write $\rho_1 := c_1/c_2$ and $\rho_2 := c_2/c_3$ for the cost ratios, so that Assumption 1 reads $\max\{\rho_1, \rho_2\} < \kappa$. When $\rho_2 < \rho_1$, so that costs rise faster near the deadline, Lemma 1 yields four types of agents. A *severe procrastinator* ($\beta_i < \rho_1\rho_2$) completes on date 3. An *impatient non-procrastinator* ($\rho_1\rho_2 < \beta_i < \rho_2$) completes on date 1, because she knows that otherwise she would wait until date 3. A *moderate procrastinator* ($\rho_2 < \beta_i < \rho_1$) completes on date 2, and a *non-procrastinator* ($\beta_i > \rho_1$) on date 1.

4.1 The last decision date

Lemma 3 (Date 2)**.** *Let $T = 3$. At h_2, player i is eager if $\beta_i > \rho_2/\kappa$, a seeker if $\kappa\rho_2 < \beta_i < \rho_2/\kappa$, and reluctant if $\beta_i < \kappa\rho_2$. Hence, if both reach date 2 without completing, they complete jointly on date 2 if one is eager and the other is not reluctant, and they complete jointly on date 3 if neither is*

[2] For example, with $T = 6$, $c = (1, 2, 4, 8, 30, 200)$, $\kappa = 0.7$ and $\beta_i = 0.2$, player i completes on date 3 alone and on date 1 with a hopeless partner.

eager and at least one is reluctant. If one is eager and the other reluctant, the eager player completes on date 2 and the reluctant one on date 3. If both are seekers, G_2 is a coordination game.

4.2 Late and moderate partners

Consider pairs in which player 1 is reluctant on date 2 ($\beta_1 < \kappa\rho_2$) and player 2 is a seeker on date 2 who would complete on date 2 if left alone ($\rho_2 < \beta_2 < \rho_2/\kappa$). If both reach date 2, player 2 then prefers to wait for her partner's company on date 3. I call player 1 the *late* partner and player 2 the *moderate* partner. The late partner is a severe procrastinator or an impatient non-procrastinator, and the moderate partner is a moderate procrastinator whenever $\beta_2 < \rho_1$. This domain contains every pair of a severe and a moderate procrastinator in which the moderate procrastinator would rather wait for company at the deadline than complete on date 2 alone. These are the pairs in which the avoidance of bad company can arise. Define

$$\mathcal{H} := \Big\{(\beta_1,\beta_2):\ \kappa\rho_1\rho_2 < \beta_1 < \min\Big\{\frac{\rho_1\rho_2}{\kappa},\,\kappa\rho_2\Big\},\ \ \rho_2 < \beta_2 < \min\Big\{\kappa\rho_1,\,\frac{\rho_2}{\kappa}\Big\}\Big\}.$$

Theorem 2 (Hide-and-seek with three periods)**.** *Let $T = 3$ and consider generic parameters.*

(a) Let $\beta_1 < \kappa\rho_2$ and $\rho_2 < \beta_2 < \rho_2/\kappa$. If both players reach date 2 without completing, they complete jointly on date 3. The equilibrium is unique and is described in Table 3. On date 1, player 1 is reluctant if $\beta_1 < \kappa\rho_1\rho_2$, a seeker if $\kappa\rho_1\rho_2 < \beta_1 < \rho_1\rho_2/\kappa$, and eager if $\beta_1 > \rho_1\rho_2/\kappa$. Player 2 is eager if $\beta_2 > \kappa\rho_1$ and an avoider if $\beta_2 < \kappa\rho_1$.

(b) In particular, the equilibrium is in mixed strategies if and only if $(\beta_1,\beta_2) \in \mathcal{H}$. In that case player i completes on date 1 with probability π_i, where

$$\pi_1 = \frac{\kappa\beta_2 c_3 - c_1}{(\kappa\beta_2 c_3 - c_1) + (\kappa c_1 - \beta_2 c_2)}, \qquad \pi_2 = \frac{c_1 - \kappa\beta_1 c_3}{(1-\kappa)(c_1 + \beta_1 c_3)}, \tag{4}$$

and the players randomize independently. A player who waits while her partner completes on date 1 then completes alone on date $\tau_i(2)$, where $\tau_1(2) = 3$ and $\tau_2(2) = 2$, and if both wait they complete jointly on date 3.

(c) $\mathcal{H}$ is nonempty if and only if $c_2^2 < \kappa c_1 c_3$.

(d) For arbitrary (β_1,β_2), every equilibrium of the three-period game has a random outcome if and only if $(\beta_1,\beta_2) \in \mathcal{H}$ or $(\beta_2,\beta_1) \in \mathcal{H}$.

Proof. (a)–(b). At h_2, player 1 is reluctant and player 2 is a seeker. Thus Lemmas 1 and 3 give the unique joint continuation and the solo costs:

$$J_1(2) = J_2(2) = \kappa c_3, \qquad S_1(2) = c_3, \quad S_2(2) = c_2.$$

At h_1, player 1 is willing to join if and only if $\beta_1 > \kappa\rho_1\rho_2$ and willing to go alone if and only if $\beta_1 > \rho_1\rho_2/\kappa$. Player 2 is willing to join if and only if $\beta_2 > \kappa\rho_1$ and is always willing to go alone, because $\beta_2 > \rho_2 > \rho_1\rho_2/\kappa$. These comparisons and Proposition 2 give every cell of Table 3. Each cell has a unique equilibrium, so the full equilibrium is unique by Lemma 2. The table's last row is feasible only if $\rho_1\rho_2/\kappa < \kappa\rho_2$, or $\rho_1 < \kappa^2$.

Mixing occurs precisely when player 1 is a seeker and player 2 an avoider; intersecting their incentive inequalities with the domain restrictions gives $\mathcal{H}$. Substituting the displayed continuation costs into (3) yields both probabilities in (4). The solo dates are $\tau_1(2) = 3$ and $\tau_2(2) = 2$; mutual waiting leads to joint completion on date 3. This proves the stated outcome description.

(c). The β_1-interval is nonempty: $\kappa\rho_1\rho_2$ is below both $\rho_1\rho_2/\kappa$ and $\kappa\rho_2$. The β_2-interval is nonempty exactly when $\rho_2 < \kappa\rho_1$, equivalent to $c_2^2 < \kappa c_1 c_3$.

(d). On $\mathcal{H}$ or its mirror image the unique equilibrium mixes at the reached date-1 history, so its outcome is random. Conversely, suppose every equilibrium has a random outcome. If G_2 had three equilibria, both players would be seekers there. Selecting joint completion on date 2 gives $J_i(2) = \kappa c_2 < S_i(2)/\kappa$. By Theorem 1(b), neither player could then be an avoider at h_1, so G_1 would have a pure equilibrium and the resulting outcome would be deterministic, a contradiction. Hence G_2 has a unique equilibrium, which is pure because avoidance is impossible at h_2. Consequently G_1 has no pure equilibrium and is hide-and-seek.

Let i be its avoider and j its seeker. By Theorem 1(b), $J_i(2) > S_i(2)/\kappa$, while $J_i(2) \leq c_3$. Thus $S_i(2) < \kappa c_3 < c_3$, which forces $S_i(2) = c_2$ and $\beta_i > \rho_2$. Player i is therefore willing to join at h_2 and cannot complete alone on date 3 after j completes on date 2. Her cost in the unique pure continuation is consequently one of $\kappa c_2, c_2, \kappa c_3$. Since $J_i(2) > c_2/\kappa > c_2$, it must be κc_3: both wait at h_2. By Lemma 3, neither is eager and at least one is reluctant. Because $\beta_i > \rho_2$, j is reluctant and i is a seeker there. Hence (β_j, β_i) belongs to the domain in (a), and (b) places it in $\mathcal{H}$. □

The example in the introduction illustrates the opposing incentives. The moderate partner would complete on date 2 if left alone, but would wait for the late partner's company on date 3 if both remained. She therefore wants to finish on date 1 only when

Table 3: Equilibrium outcomes for late–moderate pairs ($T = 3$)

	Player 2 (moderate) on date 1	
Player 1 (late) on date 1	Eager: $\beta_2 > \kappa\rho_1$	Avoider: $\beta_2 < \kappa\rho_1$
Reluctant: $\beta_1 < \kappa\rho_1\rho_2$	$(3,1)$	$(3,1)$
Seeker: $\kappa\rho_1\rho_2 < \beta_1 < \rho_1\rho_2/\kappa$	$(1,1)$	*hide-and-seek* (mixed)
Eager: $\beta_1 > \rho_1\rho_2/\kappa$	$(1,1)$	$(1,2)$

Notes: Entries are completion dates (τ_1, τ_2) in the unique equilibrium, for $\beta_1 < \kappa\rho_2$ and $\rho_2 < \beta_2 < \rho_2/\kappa$, where $\rho_1 = c_1/c_2$ and $\rho_2 = c_2/c_3$. The last row is nonempty only if $\rho_1 < \kappa^2$. When player 2 is a moderate procrastinator ($\beta_2 < \rho_1$), outcome $(3,1)$ is the avoidance of bad company, in which player 2 completes earlier than she would alone. When, in addition, player 1 is a severe procrastinator ($\beta_1 < \rho_1\rho_2$), outcome $(1,1)$ in the second row has both players completing earlier than they would alone. In the hide-and-seek cell, completion probabilities are given by (4).

the late partner does not. The late partner instead wants company: she joins on date 1 if the moderate partner completes, and otherwise waits for company on date 3. These best responses cycle, so the game has no pure-strategy equilibrium.

By part (c), hide-and-seek requires $c_2^2 < \kappa c_1 c_3$, so that the cost on the penultimate date is small relative to the costs on the first and last dates. Costs must rise slowly at first and steeply near the deadline, as they do for tasks that become much more stressful shortly before the deadline. By part (d), for any pair of present-bias factors outside $\mathcal{H}$ and its mirror image some equilibrium has a deterministic outcome, although hide-and-seek can still be played in some equilibria when there are several. Within $\mathcal{H}$, the avoider is the less present-biased player.

Corollary 2 (Nonexistence of pure-strategy equilibria)**.** *Let $T = 3$. For every pair with $\kappa\rho_1\rho_2 < \beta_1 < \rho_1\rho_2$ (a severe procrastinator) and $\rho_2 < \beta_2 < \min\{\kappa\rho_1, \rho_2/\kappa\}$ (a moderate procrastinator), the game has no equilibrium in pure strategies. For $c = (2, 3.5, 10)$, the set $\mathcal{H}$ is empty if company lowers costs by 40 percent ($\kappa = 0.6$), and $\mathcal{H} = (0.14,\, 0.245) \times (0.35,\, 0.4)$ if company lowers costs by 30 percent ($\kappa = 0.7$).*

Corollary 2 corrects a conclusion of Cerrone (2021, Proposition 1(b)), whose analysis is restricted to pure strategies. For pure strategy profiles and generic parameters, her equilibrium concept coincides with the one used here. She concludes that whenever a severe procrastinator with $\beta_1 \geq \kappa\rho_1\rho_2$ is paired with a moderate procrastinator with $\beta_2 < \rho_2/\kappa$ (in the present notation), the unique equilibrium has both completing on date 1. The pairs in Corollary 2 satisfy these conditions, but no pure-strategy equilibrium exists for them. The conclusion therefore needs the additional condition $\beta_2 > \kappa\rho_1$, under which the moderate procrastinator is willing to join on date 1. This condition holds whenever $\mathcal{H}$

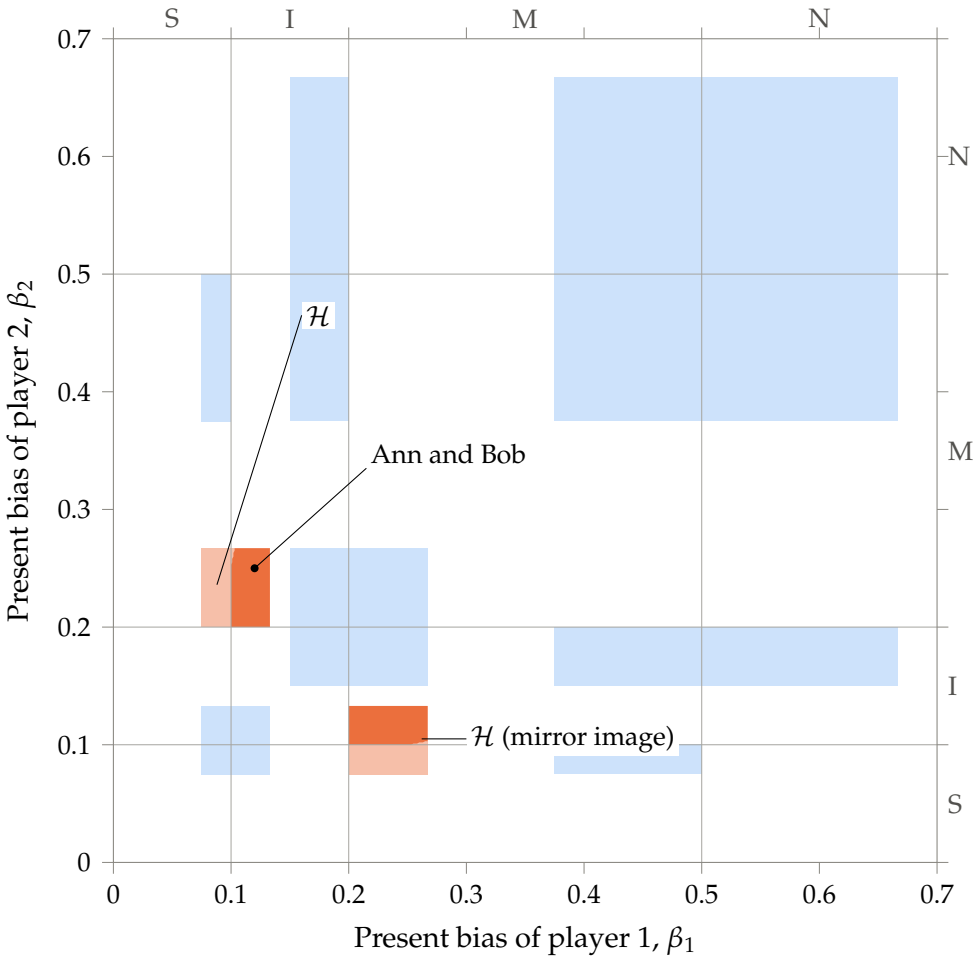


Figure 1: Equilibrium structure of the three-period game

Notes: Costs $c = (1, 2, 10)$ and $\kappa = 0.75$. White: unique equilibrium, in pure strategies (this is also the case whenever β_1 or β_2 exceeds 0.7). Blue: multiple equilibria. Light orange: unique equilibrium, in mixed strategies (hide-and-seek, the set $\mathcal{H}$ of Theorem 2 and its mirror image). Dark orange: in addition, both players are strictly worse off than if each worked alone (Theorem 3). Gray lines mark the boundaries between the four types defined at the start of Section 4: S severe procrastinator, I impatient non-procrastinator, M moderate procrastinator, N non-procrastinator. The dot is the example of the introduction.

is empty, as it is for the cost schedule $(2, 3.5, 10)$ with a 40 percent discount that she uses as an example. Her result that pairing moderate procrastinators with severe ones is efficient and stable when every severe procrastinator has $\beta_i \geq \kappa\rho_1\rho_2$ rests on the same conclusion, so it does not apply to these pairs either.

Figure 1 displays the complete equilibrium structure for $c = (1, 2, 10)$ and $\kappa = 0.75$, computed from the recursion in Appendix B. Multiple equilibria arise along the diagonal, where two players with similar present bias can coordinate on different dates, and in a few off-diagonal cells. The hide-and-seek set $\mathcal{H}$ pairs a moderate procrastinator with a severe procrastinator or an impatient non-procrastinator.

4.3 Comparative statics

By (3), each player's completion probability in the mixed equilibrium is determined by her partner's payoffs.

Proposition 5 (Comparative statics). *Let $T = 3$ and $(\beta_1, \beta_2) \in \mathcal{H}$.*

(a) The avoider's completion probability π_2 depends on β_1 but not on β_2. It is strictly decreasing in β_1, converges to 1 as $\beta_1 \downarrow \kappa\rho_1\rho_2$, and converges to 0 as $\beta_1 \uparrow \rho_1\rho_2/\kappa$ whenever $c_1 < \kappa^2 c_2$.

(b) The seeker's completion probability π_1 depends on β_2 but not on β_1, and it is strictly increasing in β_2.

(c) The probability $(1-\pi_1)(1-\pi_2)$ that both complete on date 3 is strictly increasing in β_1 and strictly decreasing in β_2.

A seeker with a higher β_1 gains more from completing with her partner on date 1. To keep her indifferent, the avoider must complete on date 1 less often, so a seeker who is more willing to complete early makes it more likely that the pair completes together at the deadline. Similarly, an avoider with a higher β_2 cares more about the future cost of waiting, and to keep her indifferent the seeker must complete on date 1 more often. In the example of the introduction, $\pi_1 = 7/9$ and $\pi_2 = 2/11$, and both complete on date 3 with probability $2/11$.

5 Welfare

5.1 Comparison with working alone

Theorem 3 (Losses from company). *Let $T = 3$, and suppose that $c_2^2 < \kappa c_1 c_3$ and $c_1 < \kappa^2 c_2$. There is a nonempty open set of present-bias pairs $(\beta_1, \beta_2) \in \mathcal{H}$ such that (i) working alone, player 1 completes on date 1 and player 2 on date 2, and (ii) in the unique equilibrium of the game both players' long-run costs strictly exceed their costs of working alone, $K_1 > c_1$ and $K_2 > c_2$. Specifically, for every $\beta_2 \in (\rho_2, \min\{\kappa\rho_1, \rho_2/\kappa\})$, (i) and (ii) hold for all $\beta_1 < \rho_1\rho_2/\kappa$ sufficiently close to $\rho_1\rho_2/\kappa$.*

In $\mathcal{H}$ the long-run costs are

$$K_1 = c_1\big[1-(1-\kappa)\pi_2\big]\Big(\pi_1 + \frac{1-\pi_1}{\beta_1}\Big), \qquad K_2 = c_1\big[1-(1-\kappa)\pi_1\big]\Big(\pi_2 + \frac{1-\pi_2}{\beta_2}\Big). \quad (5)$$

The first factor is a player's expected cost when she completes on date 1, given her partner's randomization. By indifference, her expected cost when she waits equals this amount

divided by her present-bias factor. From the long-run perspective, each player bears the full cost of the delay that her present-biased self is indifferent to. In the pairs of Theorem 3, neither player is a severe procrastinator, since alone one would complete on date 1 and the other on date 2. In Figure 1, both players are worse off than alone on almost all of the part of $\mathcal{H}$ in which player 1 is an impatient non-procrastinator. In the example of the introduction, $K_1 = 2.51 > 1$ and $K_2 = 2.78 > 2$. The condition $c_1 < \kappa^2 c_2$ is sufficient but not necessary. With the present-bias factors of the introduction and $\kappa = 0.7$, for instance, $c_1 > \kappa^2 c_2$, and the long-run costs are 2.36 and 2.50.

When the seeker is a severe procrastinator ($\beta_1 < \rho_1\rho_2$), she gains from the partnership, because every outcome costs her at most c_3, her cost of working alone, and some outcomes cost strictly less. The avoider's cost may still rise above c_2.

Losses of this kind also occur with longer horizons. With $T = 4$, $c = (1, 2, 10, 200)$, $\kappa = 0.8$ and $(\beta_1, \beta_2) = (0.11, 0.22)$, the unique equilibrium has hide-and-seek on date 1, with player 1 as the seeker and player 2 as the avoider, and if both wait they complete together on date 3. Alone, player 1 would complete on date 1 and player 2 on date 2, at costs 1 and 2. In the game their expected costs are 3.39 and 3.05.

5.2 The value of a partner

Fix a moderate procrastinator with $\beta_2 \in (\rho_2, \min\{\kappa\rho_1, \rho_2/\kappa\})$, who is an avoider whenever her partner is late, and consider how her long-run cost depends on her partner's present bias.

Proposition 6 (The value of a partner)**.** *Let $T = 3$ and $c_2^2 < \kappa c_1 c_3$, and let $\rho_2 < \beta_2 < \min\{\kappa\rho_1, \rho_2/\kappa\}$. As a function of the partner's present bias $\beta_1 \in (0, \kappa\rho_2)$, player 2's long-run cost K_2:*

(a) equals c_1 for $\beta_1 < \kappa\rho_1\rho_2$;

(b) is continuous and strictly increasing on $\left(\kappa\rho_1\rho_2, \bar{\beta}_1\right)$, where $\bar{\beta}_1 := \min\{\rho_1\rho_2/\kappa, \kappa\rho_2\}$, with $\lim_{\beta_1 \downarrow \kappa\rho_1\rho_2} K_2 = c_1\left[1 - (1-\kappa)\pi_1\right] < c_1$;

(c) if $c_1 < \kappa^2 c_2$, satisfies $\lim_{\beta_1 \uparrow \rho_1\rho_2/\kappa} K_2 = \pi_1 c_2 + (1-\pi_1)\kappa c_3 > c_2$, and equals c_2 for $\beta_1 \in (\rho_1\rho_2/\kappa, \kappa\rho_2)$.

Figure 2 illustrates. A hopeless or nearly hopeless partner is valuable to the moderate procrastinator. She completes on date 1 to avoid such a partner, and when the partner is barely willing to join her (β_1 just above $\kappa\rho_1\rho_2$) she sometimes has company on date 1. As β_1 increases within the hide-and-seek region, she completes on date 1 less often, the pair completes together on date 3 more often (Proposition 5), and her long-run cost rises.

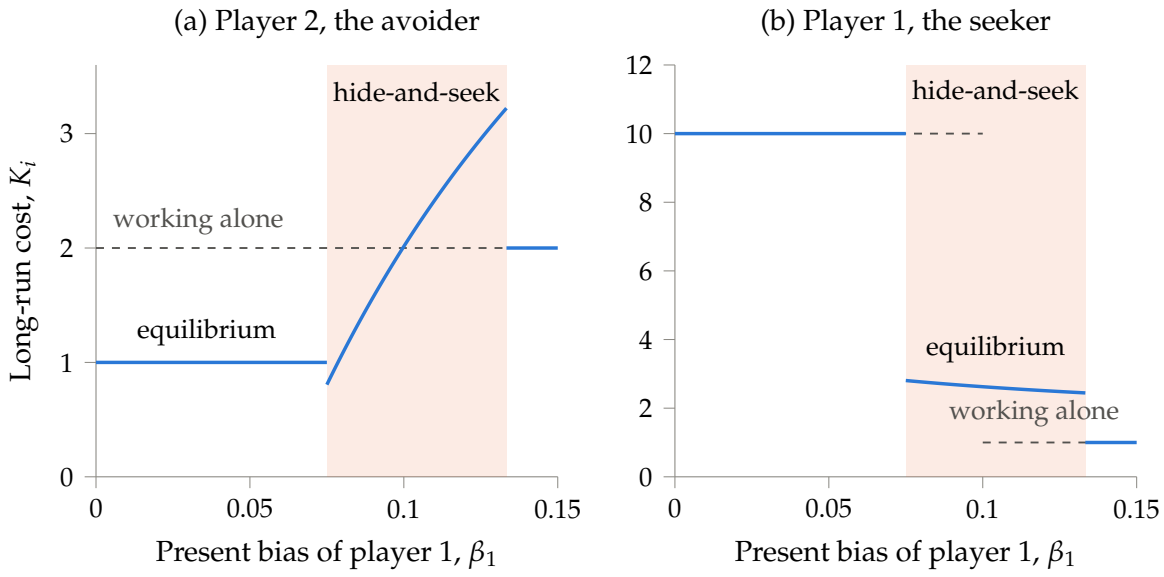


Figure 2: Long-run costs as a function of the late partner's present bias

Notes: Costs $c = (1, 2, 10)$, $\kappa = 0.75$, and $\beta_2 = 0.25$. Solid lines: expected long-run cost in the unique equilibrium. Dashed lines: cost of working alone. The shaded interval is the hide-and-seek range $(\kappa\rho_1\rho_2, \rho_1\rho_2/\kappa) = (0.075, 0.133)$. For $\beta_1 < 0.075$, player 1 is reluctant on date 1 and player 2 avoids her by completing on date 1. For $\beta_1 > 0.133$, player 1 completes on date 1 and player 2 waits until date 2.

When $c_1 < \kappa^2 c_2$, as in the figure, the hide-and-seek region ends at $\beta_1 = \rho_1\rho_2/\kappa$, where the partner becomes willing to complete on date 1 alone. Just below this threshold, her cost exceeds what she would pay alone. Just above it, the partner completes on date 1 without her, and she completes on date 2 as she would alone. As a result, her cost approaches its supremum over late partners as β_1 approaches the threshold from below, which in the example happens within the range of impatient non-procrastinators. If instead $c_1 \geq \kappa^2 c_2$, the hide-and-seek region extends to $\beta_1 = \kappa\rho_2$, the upper end of the range of late partners, and whether her cost exceeds c_2 there depends on the parameters. In both cases her cost increases with her partner's self-control throughout the hide-and-seek region.

5.3 Implications for matching

A principal who observes present bias and wants to reduce procrastination might pair moderate procrastinators with severe ones, in the hope that each completes earlier than she would alone. Pairing the moderate procrastinator of Proposition 6 with a hopeless partner or with a non-procrastinator yields a unique equilibrium with a deterministic outcome. Pairing her with a late partner who would join her early but would not complete early

alone leads to hide-and-seek, with random completion dates and possibly higher costs for both (Theorem 3), unless starts are observable (Section 6).

6 Observable starts

Hide-and-seek arises because each player must decide without knowing whether her partner has started that day. In many settings a player can instead see her partner start and join her later in the day. This section adds such observability to the model.

6.1 Mornings and afternoons

Suppose that each date t consists of two stages, a morning and an afternoon. A player who has not completed may complete in either stage. Completing on date t costs c_t, or κc_t if the partner also completes on date t, in either stage. Morning completions are observed before the afternoon. Present bias operates across dates only, so the morning and the afternoon of date t belong to the same self, who does not discount between them. The game of Sections 2 to 5 is the case in which morning completions are not observed, so that the morning stage is redundant.

A morning completion is irreversible. Interpreting it as a start requires that starting commit a player to finishing that day, as when a friend has already gone to the library or a colleague has opened a shared writing session. Announcements that do not commit the speaker would require a different model. The company benefit depends only on the completion date, so a player who joins in the afternoon receives the full discount. The extension applies to activities in which joining later preserves the value of company.

The analysis of Section 3 carries over. Solo continuations are unchanged, and at each joint history h_t the date-t selves play a two-stage game. If exactly one player completes in the morning, her partner decides in the afternoon whether to join her. If neither does, they play the simultaneous game G_t in the afternoon. Continuation costs and motives are defined as before, using the equilibrium continuation of the game with observable starts. An equilibrium's outcome is again its distribution over completion dates, so equilibria with the same outcome may differ in the timing of completions within a day.

Proposition 7 (Observable starts at a single date)**.** *Fix a joint history h_t of the game with observable starts and an equilibrium continuation from h_{t+1}, and let G_t be the corresponding simultaneous stage game. For generic parameters:*

(a) *In every equilibrium of the date-t game, each realization of date-t play that has positive probability either has both players completing on date t or results in the same completions on date t as a pure action profile that has positive probability in some Nash equilibrium of G_t.*

(b) *If G_t is a game of hide-and-seek, then in every equilibrium of the date-t game the avoider completes in the morning with probability one and the seeker completes on the same date, so both complete on date t. Compared with the Nash equilibrium of G_t, this strictly lowers the expected costs of both players' date-t selves and both players' long-run costs.*

(c) *If G_t has a unique Nash equilibrium and it is pure, the date-t game has a unique equilibrium outcome. It coincides with the outcome of G_t, except when one player is reluctant but prefers completing jointly today to the joint continuation ($\kappa c_t < \beta_i J_i(t+1)$) and the other is a seeker. In that case the reluctant player completes in the morning and the seeker completes on the same date.*

Observability thus adds only one possible outcome on date t, joint completion. In a game of hide-and-seek, an avoider who completes in the morning is sure to have company that day, because the seeker joins her, and her partner is not left to tempt her on later dates. Given that the avoider starts in the morning, the seeker is indifferent between starting in the morning and joining in the afternoon. She may mix between the two, as long as she does not start in the morning so often that the avoider would prefer to wait. In every equilibrium both complete that day and the avoider starts in the morning, and in the equilibrium in which the seeker always waits for the afternoon, the avoider leads and the seeker follows. Observability can also end joint delay when a reluctant player prefers completing jointly today to the joint continuation and her partner is a seeker. She then starts in the morning, and the seeker joins her (part (c)).

6.2 Three periods

Theorem 4 (Observable starts with three periods)**.** *Let $T = 3$ and suppose that the game with simultaneous moves has a unique equilibrium. Then, for generic parameters, the game with observable starts has a unique equilibrium outcome. It coincides with the outcome of the simultaneous game unless the simultaneous equilibrium is in mixed strategies, that is, unless (β_1, β_2) or (β_2, β_1) belongs to $\mathcal{H}$. In that case both players complete jointly on date 1, which is the best feasible outcome for both. Hence observable starts never raise either player's long-run cost, and they strictly lower both players' long-run costs in the hide-and-seek region.*

In the example of the introduction, both write on Monday and each pays 0.75 instead of 2.51 and 2.78. Bob starts in the morning, and Ann starts in the morning with some

probability $q \in [0, 2/11]$ and otherwise joins him in the afternoon. Since the avoider in $\mathcal{H}$ is the less present-biased player (Theorem 2), the player who starts in the morning for sure is the one with more self-control.

6.3 Longer horizons

With more than three periods, observability can make a player worse off, because the prospect of hide-and-seek at a later date can induce her to complete early.

Example 1 (Observable starts with four dates)**.** Let $T = 4$, $c = (1, 5, 10, 100)$, $\kappa = 0.8$ and $(\beta_1, \beta_2) = (0.12, 0.06)$. With simultaneous moves the equilibrium is unique, and player 1 completes on date 1 and player 2 on date 2, at costs 1 and 5. The joint history h_2 is off the equilibrium path. At h_2 the stage game is hide-and-seek, with player 1 as the avoider, and this makes player 1 eager on date 1. With observable starts, hide-and-seek at h_2 would end in joint completion on date 2. Anticipating this, player 1 is only a seeker on date 1 and player 2 is reluctant, so the unique equilibrium outcome is joint completion on date 2, at cost 4 each. Observability lowers player 2's cost from 5 to 4 and raises player 1's cost from 1 to 4.

In the simultaneous game, the prospect of hide-and-seek at h_2 is what makes player 1 complete on date 1. Observable starts remove this prospect and with it her reason to complete early.

7 Extensions and discussion

7.1 Naive players

Following O'Donoghue and Rabin (1999*a*), a (fully) naive player believes that her future selves will behave as if they were time-consistent. To isolate her motive, let her hold arbitrary beliefs about her partner's behavior.

Proposition 8 (Naive players)**.** *At any joint history and for any beliefs about her partner's behavior, a naive player is never an avoider. If she prefers to complete today when her partner waits, she also prefers to complete today when her partner completes.*

By Assumption 1, a time-consistent self completes the task immediately at any history. A naive self therefore believes that if she waits she will complete tomorrow, at a cost no

higher than c_{t+1}. Since she does not anticipate her own future procrastination, she has no reason to fear her partner's future company, and a pair of fully naive players never plays chicken or hide-and-seek. Because the inequalities that define avoidance are strict, avoidance should survive mild partial naiveté, in which a player slightly underestimates her future present bias (O'Donoghue and Rabin, 2001). Showing this requires a specification of each player's beliefs about her partner's beliefs, as in Flesch et al. (2026).

7.2 Identical procrastinators

Two avoiders play a game of chicken, which can arise between identical players.

Proposition 9 (Chicken). *Let $T = 3$ and $\beta_1 = \beta_2 = \beta$ with $\rho_2 < \beta < \min\{\kappa\rho_1, \rho_2/\kappa\}$. Then:*

(a) joint completion on date 2, the date each player would choose alone, is an equilibrium outcome;

(b) there are equilibria in which one player completes on date 1 and the other on date 2, each alone, with either player as the early one; and

(c) there is an equilibrium in which both randomize on date 1 and, with positive probability, both complete jointly on date 3.

With observable starts, the unique equilibrium outcome is joint completion on date 2.

Which of these equilibria arises depends on what the players expect to happen if both reach date 2. Both are then seekers, and G_2 has two pure equilibria, in which they either complete together on date 2, as each would alone, or wait together until date 3. If the players expect to complete together on date 2, neither is willing to complete on date 1. If they expect to complete together on date 3, each prefers to complete on date 1 if the other does not, but prefers to wait if the other completes, so they play chicken on date 1. In a pure equilibrium of that game, one player completes on date 1 at cost $c_1 < \kappa c_2$ and the other on date 2 at cost $c_2 > \kappa c_2$. With observable starts, joint completion on date 3 cannot be expected, because either player would start on date 2 and the other would join her.

7.3 Hide-and-seek on several dates

With longer horizons, hide-and-seek can be played on more than one date, and the players can switch roles.

Example 2 (Five dates). Let $T = 5$, $c = (1, 2, 3, 5, 20)$, $\kappa = 0.75$ and $(\beta_1, \beta_2) = (0.12, 0.28)$. Alone, player 1 would complete on date 2 and player 2 on date 1. The equilibrium is unique. On date 1, player 1 is an avoider and player 2 a seeker. If both wait, on date

Table 4: Completion dates in Example 2

(τ_1, τ_2)	$(1,1)$	$(1,4)$	$(2,1)$	$(3,3)$	$(3,4)$	$(5,3)$	$(5,5)$
Probability	0.022	0.085	0.183	0.369	0.046	0.262	0.033

Notes: Unique equilibrium of the five-period game with $c = (1, 2, 3, 5, 20)$, $\kappa = 0.75$ and $(\beta_1, \beta_2) = (0.12, 0.28)$. Hide-and-seek is played on dates 1 and 3.

2 player 1 is a seeker and player 2 is reluctant, so both wait again. On date 3 they play hide-and-seek with the roles reversed, with player 2 as the avoider. If both wait once more, they complete together on date 5. Table 4 reports the distribution of completion dates. Expected long-run costs are 7.16 and 2.96, against 2 and 1 when working alone.

Unlike in the three-period game, the first avoider here is the more present-biased player. Completion dates are also widely dispersed, although each player's environment is deterministic.

7.4 Interpreting mixed strategies

Randomization by a single self may seem unnatural. By the purification theorem of Harsanyi (1973), however, the mixed equilibrium of the hide-and-seek stage game is the limit of pure-strategy equilibria of perturbed games in which each self's costs are subject to small, privately observed shocks. Whether a player completes early then depends on how she feels that day, which her partner cannot predict. The completion probabilities can also be read as describing an outside observer's uncertainty about behavior. On either reading, the model predicts that the completion dates of pairs in $\mathcal{H}$ and its mirror image are random, with occasional joint completion at the deadline, although each player alone would complete on a predictable date.

7.5 Other extensions

With more than two players, an avoider is an agent who would rather complete today alone than continue with the group but would stay behind if the group completed today. In a large population in which each agent is negligible, no one can be followed, so hide-and-seek should disappear while avoidance of the group remains. A precise statement requires a model of how the value of company depends on the number of co-workers. With private information about present bias, waiting reveals information about one's type,

and hide-and-seek may turn into a screening problem. The choice of whether and with whom to work could also be made endogenous.

8 Conclusion

A partner's future company can tempt a sophisticated procrastinator into delay, making her want to finish early to avoid it. When the partner seeks company, their conflicting incentives produce hide-and-seek. In the three-period model, the unique equilibrium is mixed on an open parameter set that exists exactly when costs rise sufficiently sharply near the deadline. Company can then harm both partners, and greater partner self-control can raise an agent's cost. Observable, irreversible starts induce joint completion at a hide-and-seek date when later same-day joining preserves the full company benefit. This local improvement need not improve welfare with longer horizons.

The model predicts more dispersed completion dates for pairs in $\mathcal{H}$ than for the same agents working alone, and non-monotone effects of partner self-control. It also predicts that peer effects depend on whether starts can be observed, as in shared offices or on co-working platforms.

Data and code availability

The paper uses theoretical analysis and model-generated numerical examples. The accompanying replication package provides the Python scripts and instructions for reproducing the numerical examples, simulation checks, and computational figures.

Disclosure of artificial intelligence assistance

Claude Opus 5.5 was used for figure generation, code for the figures and table, the specific numerical example, and drafting the abstract, introduction, and conclusion. OpenAI GPT-6 was subsequently used to prepare this submission version, including substantive editing, shortening the abstract, condensing and auditing the central proof, and preparing and checking the submission files.

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A Proofs

Throughout, for a joint history h_t and a fixed equilibrium continuation, write $S_i := S_i(t+1)$ and $J_i := J_i(t+1)$, and recall that player i is willing to join if $\kappa c_t < \beta_i S_i$ and willing to go alone if $c_t < \beta_i J_i$. A player alone from date $t+1$ completes on some date $s \geq t+1$ at cost c_s, so $S_i(t+1) \geq c_{t+1}$. Every realized cost in a continuation from h_{t+1} is at least κc_{t+1}, so $J_i(t+1) \geq \kappa c_{t+1} > c_t$ by Assumption 1.

A.1 Genericity

Lemma A.1. *There is a Lebesgue-null set $\mathcal{P}_0$ of parameters $(c, \kappa, \beta_1, \beta_2)$ such that, outside $\mathcal{P}_0$: (i) $c_s \neq \beta_i c_{s'}$ and $c_s \neq \beta_i \kappa c_{s'}$ for all i and $s < s'$; (ii) for every equilibrium, every joint history h_t with $t < T$ and every i, $\kappa c_t \neq \beta_i S_i(t+1)$, $c_t \neq \beta_i J_i(t+1)$ and $\kappa c_t \neq \beta_i J_i(t+1)$; and (iii) from every joint history there are finitely many equilibrium continuations, and hence finitely many equilibrium continuation costs.*

Proof. The parameter space is a subset of $\mathbb{R}^{T+3}$ with nonempty interior, defined by finitely many polynomial inequalities. Proceed by backward induction on t. Play from h_{t+1} on is determined by the solo costs $S_i(s)$ for $s \geq t+1$ and the set of equilibrium continuation costs $J(t+1)$, and these depend only on $(c_{t+1}, \ldots, c_T, \kappa, \beta_1, \beta_2)$, not on c_t or earlier costs. Suppose that, for these parameters, (i)–(iii) hold at all dates after t, so that there are finitely many continuation costs $J(t+1)$. The equalities to be excluded at date t are $c_t = \beta_i c_{s'}$, $c_t = \beta_i \kappa c_{s'}$, $\kappa c_t = \beta_i S_i(t+1)$, and $c_t = \beta_i J_i(t+1)$ or $\kappa c_t = \beta_i J_i(t+1)$ for each of the finitely many $J(t+1)$. Each is an affine equation in c_t with nonzero slope, so it holds for at most one value of c_t. Hence, for fixed values of the other parameters, the set of c_t violating some condition at date t is finite. The continuation costs are rational functions of the parameters on each of finitely many semialgebraic pieces, so all sets involved are semialgebraic and hence measurable; by Fubini's theorem, the set of parameters for which (i)–(ii) hold after t but fail at t has measure zero. Outside it, every stage game G_t has strict preferences against pure actions and hence finitely many Nash equilibria (Proposition 2), which gives (iii) at t. Taking the union over t yields the null set $\mathcal{P}_0$. □

The same conclusions hold for the game with observable starts, with (iii) stated for continuation costs; this is shown after the proof of Proposition 7 below.

A.2 Proofs for Sections 2 and 3

Proof of Lemma 1. At a solo history player j has completed, so player i pays c_s if she completes on date s. Her remaining selves play a finite game of perfect information. Self T must complete. Given that her future selves complete on date $\tau_i(s+1)$, self $s < T$ compares c_s with $\beta_i c_{\tau_i(s+1)}$ and, by Lemma A.1(i), strictly prefers one action. Backward induction yields the unique equilibrium described. □

Proof of Lemma 2. At a solo history only one player moves, and her payoffs are those of Lemma 1; hence any equilibrium prescribes the solo continuation there. Consider a joint history h_t, $t < T$, and let p_j be the probability with which self t of player j completes at h_t. If self t of player i completes, her expected cost is $p_j \kappa c_t + (1 - p_j)c_t$. If she waits, then with probability p_j player j completes and i continues alone, paying $S_i(t+1)$; with probability $1 - p_j$ play reaches h_{t+1}, where her expected realized cost is $J_i(t+1)$. Since all costs after date t are discounted by β_i, her expected cost from waiting is $\beta_i[p_j S_i(t+1) + (1 - p_j)J_i(t+1)]$. These are the expected costs of the stage game G_t in (2). Thus selves t best respond at h_t if and only if their completion probabilities form a Nash equilibrium of G_t. Conversely, a profile satisfying (i) and (ii) has every self best responding at every history, because solo and joint histories are the only histories at which selves move. □

Proof of Proposition 1. By Lemma A.1, in every stage game each player strictly prefers one action against each pure action of her partner. By Proposition 2, such a 2×2 game has one or three Nash equilibria. An equilibrium is constructed by choosing a Nash equilibrium of G_{T-1} (given $J(T) = (\kappa c_T, \kappa c_T)$), computing $J(T-1)$, choosing a Nash equilibrium of G_{T-2}, and so on, and prescribing solo continuations at solo histories; by Lemma 2 this is an equilibrium. Every equilibrium arises in this way, and it is determined by its $T - 1$ choices, so there are at most 3^{T-1} equilibria. □

Proof of Proposition 2. Player i's best response to Y is Y if and only if she is willing to join, and her best response to N is Y if and only if she is willing to go alone; by genericity, these best responses are unique. (a) An eager (reluctant) player has the strictly dominant action Y (N), and her partner has a unique best response to it. (b) Two seekers: each best response matches the partner's action, so (Y,Y) and (N,N) are the pure equilibria. (c) Two avoiders: each best response mismatches, so (Y,N) and (N,Y) are the pure equilibria. (d) A seeker and an avoider: checking the four profiles, the avoider wants to deviate from (Y,Y) and (N,N) and the seeker from (Y,N) and (N,Y), where the first action is the seeker's; so there is no pure equilibrium. In any equilibrium in which a player mixes she must be indifferent, which, against a pure action of the partner, is ruled out by genericity; hence

any equilibrium other than the pure ones is completely mixed. In a completely mixed equilibrium, player i is indifferent:

$$\pi_j \kappa c_t + (1-\pi_j)c_t = \beta_i\big[\pi_j S_i + (1-\pi_j)J_i\big],$$

which yields (3). For a seeker, $c_t - \beta_i J_i > 0$ and $\beta_i S_i - \kappa c_t > 0$; for an avoider, both are negative. In cases (b)–(d), each player is a seeker or an avoider, so (3) lies in $(0,1)$ for both players and defines a completely mixed equilibrium, which is unique. In case (a), a player with a dominant action cannot be made indifferent, so there is no completely mixed equilibrium. □

Proof of Theorem 1. (a) An avoider is not willing to join: $\beta_i S_i(t+1) < \kappa c_t$. Hence $\beta_i < \kappa c_t / S_i(t+1) \leq \kappa c_t / c_{t+1} < \kappa$. If $\beta_i = 1$, then by Lemma 1 $\tau_i(s) = s$ for every s, because costs increase; so $S_i(t+1) = c_{t+1} > \kappa c_t$ and player i is willing to join. Moreover $J_i(t+1) \geq \kappa c_{t+1} > c_t$, so she is willing to go alone. She is therefore eager at every joint history, and at h_1 she completes (Proposition 2(a)).

(b) An avoider satisfies $\beta_i J_i(t+1) > c_t$ and $\beta_i S_i(t+1) < \kappa c_t$. Hence $J_i(t+1) > c_t/\beta_i > S_i(t+1)/\kappa$. Suppose that, in every outcome of the continuation from h_{t+1} that has positive probability, player i completes on some date $d \leq \tau_i(t+1)$. Then each such outcome costs her at most $c_d \leq c_{\tau_i(t+1)} = S_i(t+1)$, so $J_i(t+1) \leq S_i(t+1) < S_i(t+1)/\kappa$, a contradiction.

(c) At $t = T-1$, $J_i(T) = \kappa c_T < c_T = S_i(T) < S_i(T)/\kappa$, which contradicts (b). Hence $t \leq T-2$. If $T = 2$, the only joint history at which players choose is $h_1 = h_{T-1}$, so there are no avoiders, and by Proposition 2 chicken and hide-and-seek require an avoider. □

Proof of Proposition 3. Suppose both players complete on date d with probability one in some equilibrium. Then h_d is reached with probability one and (Y,Y) is played there, so (Y,Y) is a Nash equilibrium of G_d; if $d < T$ this requires $\kappa c_d < \beta_i S_i(d+1)$ for both i, which is (i). For every $t < d$, h_t is reached and (N,N) is played; the continuation from h_{t+1} is joint completion on date d, so $J_i(t+1) = \kappa c_d$, and (N,N) is a Nash equilibrium of G_t if and only if $c_t > \kappa\beta_i c_d$ for both i. Applied to $t = 1$ this gives (ii). Conversely, suppose (i) and (ii) hold. Construct a profile that prescribes (N,N) at h_t for $t < d$, (Y,Y) at h_d, any Nash equilibrium of G_t at the joint histories h_t with $t > d$ (computed backward as in Proposition 1), and solo continuations elsewhere. By (i), (Y,Y) is a Nash equilibrium of G_d. For $t < d$, $J_i(t+1) = \kappa c_d$ and $c_t \geq c_1 > \kappa\beta_i c_d$ by (ii), so (N,N) is a Nash equilibrium of G_t. By Lemma 2 the profile is an equilibrium. □

Proof of Corollary 1. (a) Let $d = \tau_1(1) = \tau_2(1)$. If $d < T$, self d of each player completes when alone ($\tau_i(d) = d$), so $c_d < \beta_i c_{\tau_i(d+1)} = \beta_i S_i(d+1)$, which implies $\kappa c_d < \beta_i S_i(d+1)$. If $d > 1$, self 1 of each player waits when alone and her future selves complete on date d, so $c_1 > \beta_i c_d > \kappa\beta_i c_d$. Apply Proposition 3. Identical players have identical solo dates.

(b) The first statement is Proposition 3 with $d = T$. If $c_1 < \beta_i c_T < c_1/\kappa$, then, by Assumption 1, $c_s \geq c_2 > c_1/\kappa > \beta_i c_T$ for all $s \geq 2$, so when alone every self $s \geq 2$ waits and $\tau_i(2) = T$; self 1 completes because $c_1 < \beta_i c_T$. Thus $\tau_i(1) = 1$, while $\beta_i < c_1/(\kappa c_T)$.

(c) The first statement is Proposition 3 with $d = 1$. With $T = 3$, $S_i(2) = c_3$ if $\beta_i < \rho_2$ and $S_i(2) = c_2$ if $\beta_i > \rho_2$. The condition $\kappa c_1 < \beta_i S_i(2)$ therefore reads $\beta_i > \kappa\rho_1\rho_2$ if $\beta_i < \rho_2$ and $\beta_i > \kappa\rho_1$ if $\beta_i > \rho_2$. Since $\kappa\rho_1\rho_2 < \rho_2 < \kappa\rho_1$, the last inequality being equivalent to $c_2^2 < \kappa c_1 c_3$, the set is $(\kappa\rho_1\rho_2, \rho_2) \cup (\kappa\rho_1, 1]$. □

Proof of Proposition 4. Every realized cost is at most c_T. At any joint history, $\beta_j S_j(t+1) \leq \beta_j c_T < \kappa c_1 \leq \kappa c_t$ and $\beta_j J_j(t+1) \leq \beta_j c_T < \kappa c_1 < c_t$, so player j is reluctant at every joint history of every equilibrium and waits there (Proposition 2(a)). When alone, $c_s \geq c_1 > \beta_j c_T$ for all s, so $\tau_j(s) = T$. Hence j completes on date T. Given that j waits at every joint history, player i completes at h_t if and only if $c_t < \beta_i J_i(t+1)$. I show by backward induction that $J_i(t) = \hat{c}_{\hat{\tau}_i(t)}$, where $\hat{\tau}_i(t)$ is the solo completion date from t under $\hat{c}$. For $t = T$, $J_i(T) = \kappa c_T = \hat{c}_T$. If $J_i(t+1) = \hat{c}_{\hat{\tau}_i(t+1)}$, then i completes at h_t if and only if $\hat{c}_t = c_t < \beta_i \hat{c}_{\hat{\tau}_i(t+1)}$, that is, if and only if $\hat{\tau}_i(t) = t$, in which case she pays $c_t = \hat{c}_t$ because j waits; otherwise $J_i(t) = J_i(t+1) = \hat{c}_{\hat{\tau}_i(t)}$. Hence i completes on $\hat{\tau}_i(1)$. Every stage game has a unique Nash equilibrium because j is reluctant, so the equilibrium outcome is unique. □

A.3 Proofs for Sections 4 and 5

Proof of Lemma 3. With $T = 3$, $S_i(3) = c_3$ and $J_i(3) = \kappa c_3$. Player i is willing to join at h_2 if and only if $\kappa c_2 < \beta_i c_3$, that is, $\beta_i > \kappa\rho_2$, and willing to go alone if and only if $c_2 < \beta_i \kappa c_3$, that is, $\beta_i > \rho_2/\kappa$. Since $\kappa\rho_2 < \rho_2/\kappa$, a player willing to go alone is willing to join, so there are no avoiders and the motives are as stated. The equilibria of G_2 follow from Proposition 2. If one player is eager and the other is not reluctant, the eager player completes and the other's best response to Y is Y. If one is eager and the other reluctant, the outcome is (Y, N), and the reluctant player completes alone on date 3. If neither is eager and at least one is reluctant, the reluctant player waits and the other's best response to N is N. □

The complete proof of Theorem 2 is given in the main article.

Proof of Corollary 2. Since $\kappa < 1$ and $\rho_1 < \kappa$, $\rho_1\rho_2 < \min\{\rho_1\rho_2/\kappa, \kappa\rho_2\}$, so the pairs described belong to $\mathcal{H}$. Player 1 is a severe procrastinator ($\beta_1 < \rho_1\rho_2$) and player 2 a moderate one ($\rho_2 < \beta_2 < \kappa\rho_1 < \rho_1$). By Theorem 2(b) the unique equilibrium is mixed. For $c = (2, 3.5, 10)$, $c_2^2 = 12.25$, while $\kappa c_1 c_3 = 12$ for $\kappa = 0.6$ and 14 for $\kappa = 0.7$; with $\kappa = 0.7$, $\rho_1 = 4/7$ and $\rho_2 = 0.35$ give $\mathcal{H} = (0.14, \min\{0.2857, 0.245\}) \times (0.35, \min\{0.4, 0.5\})$. Finally, the conditions attributed to Cerrone (2021) in the text are translated from her notation as follows. She labels the severe and moderate procrastinators A and B, and her company parameter is $1 - \kappa$ in the present notation. Her conditions $\beta_A \geq (1-\kappa)c_1/c_3$ and $\beta_B < c_2/[(1-\kappa)c_3]$ are therefore $\beta_1 \geq \kappa\rho_1\rho_2$ and $\beta_2 < \rho_2/\kappa$, and the additional condition $\beta_2 > \kappa\rho_1$ reads $\beta_B > (1-\kappa)c_1/c_2$ in her notation. □

Proof of Proposition 5. (a) By (4), π_2 depends only on (β_1, c, κ). In $\mathcal{H}$ its numerator $c_1 - \kappa\beta_1 c_3$ is positive and strictly decreasing in β_1, and its denominator $(1-\kappa)(c_1 + \beta_1 c_3)$ is positive and strictly increasing, so π_2 is strictly decreasing. At $\beta_1 = \kappa\rho_1\rho_2 = \kappa c_1/c_3$, $\pi_2 = c_1(1-\kappa^2)/[(1-\kappa)(1+\kappa)c_1] = 1$; at $\beta_1 = \rho_1\rho_2/\kappa = c_1/(\kappa c_3)$ the numerator vanishes. If $c_1 < \kappa^2 c_2$, then $\rho_1\rho_2/\kappa < \kappa\rho_2$, so $\rho_1\rho_2/\kappa$ is the upper endpoint of the β_1-interval of $\mathcal{H}$. (b) Write $\pi_1 = A/(A+B)$ with $A = \kappa\beta_2 c_3 - c_1 > 0$ and $B = \kappa c_1 - \beta_2 c_2 > 0$ in $\mathcal{H}$. Then

$$\frac{\partial \pi_1}{\partial \beta_2} = \frac{\kappa c_3 B + c_2 A}{(A+B)^2} = \frac{c_1(\kappa^2 c_3 - c_2)}{(A+B)^2} > 0,$$

which is positive because $A, B > 0$. (c) Follows from (a) and (b), since $\pi_1, \pi_2 \in (0,1)$. □

Proof of Theorem 3. Let $X_1 := \pi_2\kappa c_1 + (1-\pi_2)c_1 = c_1[1-(1-\kappa)\pi_2]$ be player 1's expected realized cost when she completes on date 1. When she waits, her expected realized cost is $\pi_2 S_1(2) + (1-\pi_2)J_1(2)$, which by her indifference equals X_1/β_1. The players randomize independently, so $K_1 = \pi_1 X_1 + (1-\pi_1)X_1/\beta_1$; similarly for K_2, which gives (5). Assume $c_1 < \kappa^2 c_2$, so that the β_1-interval of $\mathcal{H}$ is $(\kappa\rho_1\rho_2, \rho_1\rho_2/\kappa)$, and fix $\beta_2 \in (\rho_2, \min\{\kappa\rho_1, \rho_2/\kappa\})$, which is nonempty because $c_2^2 < \kappa c_1 c_3$; then $\pi_1 \in (0,1)$ is fixed. (i) For $\beta_1 \in (\rho_1\rho_2, \rho_1\rho_2/\kappa)$, which lies below ρ_2 because $\rho_1 < \kappa$, player 1 is an impatient non-procrastinator and completes on date 1 alone, at cost c_1; since $\rho_2 < \beta_2 < \kappa\rho_1 < \rho_1$, player 2 is a moderate procrastinator and completes on date 2 alone, at cost c_2. (ii) As $\beta_1 \uparrow \rho_1\rho_2/\kappa$, $\pi_2 \to 0$ (Proposition 5), so $X_1 \to c_1$ and

$$K_1 \to c_1\Big(\pi_1 + \frac{1-\pi_1}{\rho_1\rho_2/\kappa}\Big) > c_1,$$

$$K_2 \to \frac{X_2}{\beta_2} = \pi_1 S_2(2) + (1-\pi_1)J_2(2) = \pi_1 c_2 + (1-\pi_1)\kappa c_3 > c_2,$$

where the first inequality uses $\pi_1 < 1$ and $\rho_1\rho_2/\kappa < 1$, and the second uses $\kappa c_3 > c_2$ (Assumption 1) and $\pi_1 < 1$. By continuity of K_1 and K_2 in β_1, there is $\varepsilon > 0$ such that $K_1 > c_1$ and $K_2 > c_2$ for all $\beta_1 \in (\rho_1\rho_2/\kappa - \varepsilon, \rho_1\rho_2/\kappa)$, and ε can be chosen so that these β_1 exceed $\rho_1\rho_2$. The set of $(\beta_1, \beta_2) \in \mathcal{H}$ with $\beta_1 > \rho_1\rho_2$, $K_1 > c_1$ and $K_2 > c_2$ is defined by strict inequalities between continuous functions on an open set, so it is open, and it is nonempty by the argument above. □

Proof of Proposition 6. (a) If $\beta_1 < \kappa\rho_1\rho_2$, Theorem 2 gives the outcome $(3, 1)$, so $K_2 = c_1$. (b) On $(\kappa\rho_1\rho_2, \bar{\beta}_1)$, $(\beta_1, \beta_2) \in \mathcal{H}$ and $K_2 = X_2[\pi_2 + (1 - \pi_2)/\beta_2]$, where $X_2 = c_1[1 - (1 - \kappa)\pi_1]$ does not depend on β_1. Since $\beta_2 < 1$, K_2 is strictly decreasing in π_2, which is continuous and strictly decreasing in β_1 (Proposition 5). Hence K_2 is continuous and strictly increasing. As $\beta_1 \downarrow \kappa\rho_1\rho_2$, $\pi_2 \to 1$ and $K_2 \to X_2 < c_1$. (c) If $c_1 < \kappa^2 c_2$, then $\bar{\beta}_1 = \rho_1\rho_2/\kappa$, and the limit is computed in the proof of Theorem 3. For $\beta_1 \in (\rho_1\rho_2/\kappa, \kappa\rho_2)$, player 1 is eager and player 2 an avoider at h_1, so the outcome is $(1, 2)$ and $K_2 = c_2$. □

A.4 Proofs for Sections 6 and 7

In the game with observable starts, fix a joint history h_t and the continuation from h_{t+1}. If exactly one player completes in the morning, her partner completes in the afternoon if and only if she is willing to join. Let $\iota_i = 1$ if player i is willing to join and $\iota_i = 0$ otherwise. If neither completes in the morning, the players play G_t in the afternoon; let V_i denote player i's expected cost in the selected Nash equilibrium of G_t (as evaluated by self t). The morning game therefore has the following costs for player i:

	j completes	j waits
i completes	κc_t	$\iota_j\, \kappa c_t + (1 - \iota_j)\, c_t$
i waits	$\iota_i\, \kappa c_t + (1 - \iota_i)\, \beta_i S_i$	V_i

(A.1)

Since V_i is a Nash equilibrium payoff, it equals the lower of i's two expected costs in G_t against her partner's equilibrium action.

Proof of Proposition 7. (a) Consider an equilibrium of the date-t game and a realization of date-t play with positive probability. If both complete in the morning, or one completes in the morning and the other joins, both complete on date t. If neither completes in the morning, afternoon play is drawn from a Nash equilibrium of G_t. It remains to consider the case in which player i completes in the morning, j waits in the morning and does not join; then j is not willing to join, so N is j's best response to Y in G_t. I claim that i is willing

to go alone, so that (Y,N) is a Nash equilibrium of G_t. Suppose not. Then $V_i < c_t$: if j completes with positive probability in the afternoon equilibrium, V_i is at most i's expected cost from completing, which is below c_t; otherwise $V_i \leq \beta_i J_i < c_t$. Let $p_j < 1$ be the probability that j completes in the morning. By (A.1), i's expected cost from completing in the morning is $p_j \kappa c_t + (1-p_j)c_t$, while waiting costs at most $p_j \kappa c_t + (1-p_j)V_i$, which is strictly lower. So i does not complete in the morning, a contradiction.

(b) Let a be the avoider and s the seeker. In the unique equilibrium of G_t, $V_a = \pi_s \kappa c_t + (1-\pi_s)c_t$ and $V_s = \pi_a \kappa c_t + (1-\pi_a)c_t$ with $\pi_a, \pi_s \in (0,1)$, so $V_a, V_s \in (\kappa c_t, c_t)$. By (A.1), a's cost from completing in the morning is κc_t whatever s does, because s is willing to join; waiting costs $\beta_a S_a < \kappa c_t$ if s completes and $V_a > \kappa c_t$ if s waits. s's cost from completing in the morning is κc_t if a completes and c_t if a waits, because a is not willing to join; waiting costs κc_t if a completes and $V_s < c_t$ if a waits. Let p be the probability that a completes in the morning. If $p < 1$, s strictly prefers to wait; but then a strictly prefers to complete, so $p = 1$, a contradiction. Hence $p = 1$, and s completes on date t, in the morning or by joining in the afternoon. Given that a completes in the morning, s is indifferent between her morning actions. If s completes in the morning with probability q, the avoider weakly prefers completing in the morning to waiting if and only if $\kappa c_t \leq q\beta_a S_a + (1-q)V_a$, that is, if and only if

$$q \leq \bar{q} := \frac{V_a - \kappa c_t}{V_a - \beta_a S_a} = \frac{(1-\kappa)c_t}{\beta_a (J_a - S_a)},$$

where the second equality uses the avoider's indifference in the equilibrium of G_t. Since $\beta_a S_a < \kappa c_t < V_a$, $\bar{q} \in (0,1)$. The equilibria of the date-t game are therefore exactly the profiles in which a completes in the morning, s completes in the morning with some probability $q \in [0, \bar{q}]$, and s joins in the afternoon otherwise. With $q = 0$, the avoider leads and the seeker follows. In the introductory example, $\bar{q} = (1-0.75)/[0.25\,(7.5-2)] = 2/11$.

Both date-t selves pay $\kappa c_t < \min\{V_a, V_s\}$. In the equilibrium of G_t every action profile has positive probability, and every profile other than (Y,Y) gives each player a realized cost of c_t, $S_i(t+1) \geq c_{t+1}$ or $J_i(t+1) \geq \kappa c_{t+1}$, each strictly above κc_t by Assumption 1. Hence both long-run costs are strictly lower when both complete on date t.

(c) If G_t has a unique Nash equilibrium that is pure, then by Proposition 2 some player has a dominant action. *Case 1: some player i is eager.* In G_t, i completes and j joins if and only if she is willing to join, so $V_i = \kappa c_t$ if j is willing to join and $V_i = c_t$ otherwise; by (A.1), i is indifferent between her morning actions against either action of j. If j is willing to join, every morning profile leads to joint completion on date t, the outcome of G_t. If not, waiting is strictly dominant for j in the morning: completing costs her κc_t (because i would join), while waiting costs $\beta_j S_j < \kappa c_t$ whether i completes in the morning or, in the afternoon,

plays her equilibrium action Y. So j waits, i completes on date t, and j continues alone, as in G_t. *Case 2: some player i is reluctant* (and, by Case 1, no player is eager). If j is willing to go alone, then in G_t player i waits and j completes, so $V_i = \beta_i S_i$ and $V_j = c_t$; in the morning, waiting is strictly dominant for i, since completing costs at least $\kappa c_t > \beta_i S_i$, and j is indifferent, because i would not join her; every equilibrium yields the outcome of G_t. If j is not willing to go alone, then in G_t both wait, $V_i = \beta_i J_i$ and $V_j = \beta_j J_j$. If j is reluctant, waiting is strictly dominant for both in the morning, and the outcome is that of G_t. If j is a seeker, then by (A.1) j's cost from completing in the morning is κc_t if i completes and c_t if i waits, while waiting costs κc_t if i completes and $\beta_j J_j < c_t$ if i waits; and i strictly prefers to wait if j completes, while if j waits she prefers to complete if and only if $\kappa c_t < \beta_i J_i$. If $\kappa c_t > \beta_i J_i$, waiting is strictly dominant for i, j then waits, and the outcome is that of G_t. If $\kappa c_t < \beta_i J_i$, let a be the probability that i completes in the morning: if $a < 1$, j strictly prefers to wait, and then i strictly prefers to complete, a contradiction; so $a = 1$, j joins, and both complete on date t. □

Genericity with observable starts. The conclusions of Lemma A.1 also hold for the game with observable starts, with (iii) replaced by finiteness of the set of equilibrium continuation-cost vectors. The argument is the same backward induction. Suppose that the set of continuation-cost vectors from h_{t+1} is finite, each vector being a rational function of the parameters on each of finitely many semialgebraic pieces; this holds for $t+1 = T$, where $J(T) = (\kappa c_T, \kappa c_T)$. As in the proof of Lemma A.1, exclude the finitely many values of c_t at which one of the comparisons in (i)–(ii) holds with equality. Fix a continuation from h_{t+1} and an equilibrium of G_t to be played in the afternoon, and let V_i be self t's expected cost in it, so that the morning game is (A.1). If G_t has a unique Nash equilibrium, the arguments in the proof of Proposition 7(b)–(c), which use only the strict inequalities just imposed, give a unique continuation-cost vector at h_t. Two cases remain.

Two seekers. Every morning profile other than mutual waiting leads to joint completion on date t, because the partner joins. If the afternoon equilibrium is (Y, Y), so does mutual waiting. If it is completely mixed, then $V_i > \kappa c_t$ for both players, so in every morning equilibrium at least one player completes with probability one. If it is (N, N), then $V_i = \beta_i J_i \neq \kappa c_t$; each player is indifferent between her morning actions when her partner completes and has a strict preference when her partner waits, so in every morning equilibrium either at least one player completes with probability one or both wait with probability one. The continuation-cost vector is therefore $(\kappa c_t, \kappa c_t)$ or $J(t+1)$.

Two avoiders. If the afternoon equilibrium is pure, the player who waits in it strictly prefers to wait in the morning, and the other is indifferent between completing in the

morning and completing in the afternoon, so the completion dates are those of the afternoon equilibrium. If the afternoon equilibrium is mixed, then $V_i < c_t$ and $\beta_i S_i < \kappa c_t$ for both players, so waiting is strictly dominant in the morning and the afternoon outcome is again preserved.

In every case there are finitely many continuation-cost vectors at h_t, each a rational function of the parameters on each of finitely many semialgebraic pieces, and the Fubini argument in the proof of Lemma A.1 applies.

Proof of Theorem 4. Suppose the simultaneous game has a unique equilibrium. Then G_2, whose continuation cost $J(3) = (\kappa c_3, \kappa c_3)$ is common to both games, has a unique Nash equilibrium; it is pure, because there are no avoiders at h_2 (Lemma 3). By Proposition 7(c), the date-2 outcome with observable starts coincides with that of G_2 unless one player is reluctant with $\kappa c_2 < \beta_i J_i(3) = \kappa \beta_i c_3$; but a reluctant player has $\beta_i < \kappa \rho_2 < \rho_2$, so $\kappa c_2 > \kappa \beta_i c_3$. Hence $J(2)$ is the same in both games, and so is $S(2)$, by Lemma 1. The date-1 stage game G_1 is therefore the same in both games, and it has a unique Nash equilibrium, which is either pure or, by Proposition 2, the equilibrium of a hide-and-seek game. By Proposition 7(b)–(c), the date-1 outcome with observable starts is unique, and it coincides with that of G_1 unless G_1 is a game of hide-and-seek, or one player i is reluctant with $\kappa c_1 < \beta_i J_i(2)$ and the other is a seeker. The second case cannot occur. Such a player would satisfy $\beta_i J_i(2) > \kappa c_1 > \beta_i S_i(2)$, so $J_i(2) > S_i(2) \geq c_2$; since play on date 2 is pure, $J_i(2) \in \{\kappa c_2, c_2, c_3, \kappa c_3\}$, which leaves c_3 and κc_3. If $J_i(2) = c_3$, player i completes alone on date 3 after her partner completes on date 2, so she is not willing to join at h_2, hence reluctant there; then $\beta_i < \kappa \rho_2 < \rho_2$ and $S_i(2) = c_3 = J_i(2)$, a contradiction. If $J_i(2) = \kappa c_3$, then $S_i(2) < \kappa c_3 < c_3$ forces $S_i(2) = c_2$, i.e. $\beta_i > \rho_2$; but i is not willing to go alone at h_1, so $c_1 > \kappa \beta_i c_3$, i.e. $\beta_i < \rho_1 \rho_2 / \kappa < \rho_2$, a contradiction. In the remaining exceptional case each player pays κc_1, the lowest feasible realized cost. By Theorem 2(d), the simultaneous equilibrium is mixed if and only if (β_1, β_2) or (β_2, β_1) belongs to $\mathcal{H}$; in that case Proposition 7(b) shows that both long-run costs fall strictly. □

Proof of Proposition 8. A naive self t evaluates waiting by assuming that, at every later date, her selves minimize realized cost given her partner's behavior. Under Assumption 1 such a self completes immediately at any history: completing on date s costs at most c_s, while completing on a later date s' costs at least $\kappa c_{s'} > c_{s'-1} \geq c_s$. Hence a naive self t believes that, if she waits, she will complete on date $t+1$, at cost c_{t+1} if her partner has completed and at most c_{t+1} otherwise. If she prefers to complete today when her partner waits, then $c_t < \beta_i \times (\text{perceived cost}) \leq \beta_i c_{t+1}$. If her partner completes today, waiting is perceived to cost $\beta_i c_{t+1} > c_t > \kappa c_t$, so she prefers to complete today as well. □

Proof of Proposition 9. Since $\kappa\rho_2 < \rho_2 < \beta < \rho_2/\kappa$, both players are seekers at h_2 (Lemma 3), and G_2 has the equilibria (Y,Y), (N,N) and a mixed one. Also $S_i(2) = c_2$ because $\beta > \rho_2$. If (Y,Y) is selected at h_2, then $J_i(2) = \kappa c_2$; at h_1, neither player is willing to join, since $\beta c_2 < \kappa\rho_1 c_2 = \kappa c_1$, nor willing to go alone, since $\beta\kappa c_2 < \beta c_2 < \kappa c_1 < c_1$. Both are reluctant, both wait, and they complete jointly on date 2, which proves (a). If (N,N) is selected at h_2, then $J_i(2) = \kappa c_3$; at h_1 neither is willing to join, as before, but both are willing to go alone, because $c_1 < \beta\kappa c_3$ is equivalent to $\beta > \rho_1\rho_2/\kappa$, which holds since $\beta > \rho_2 > \rho_1\rho_2/\kappa$. Both are avoiders, and G_1 is a game of chicken with equilibria (Y,N), (N,Y) and a completely mixed one. In (Y,N), player 1 completes on date 1 and player 2 completes alone on $\tau_2(2) = 2$, and (N,Y) is symmetric, which proves (b). In the mixed equilibrium both wait with positive probability, and they then complete jointly on date 3, which proves (c). With observable starts, at h_2 each player prefers joint completion on date 2 to joint completion on date 3, since $\kappa c_2 < \beta\kappa c_3$ is equivalent to $\beta > \rho_2$. In the morning, if her partner waits, a player who completes is joined, at cost κc_2, while waiting costs V_i, which exceeds κc_2 in each equilibrium of G_2 other than (Y,Y); hence every equilibrium of the date-2 game ends in joint completion on date 2. Then $J_i(2) = \kappa c_2$, both players are reluctant at h_1, and by Proposition 7(c) both wait on date 1, since neither prefers joint completion on date 1 to joint completion on date 2 ($\beta\kappa c_2 < \kappa c_1$). □

B The three-period game: computing the equilibrium correspondence

For $T = 3$, the equilibrium set for arbitrary (β_1, β_2) can be computed in three steps. Figure 1 was computed in this way.

Step 1 (date 2). By Lemma 3, each player is reluctant ($\beta_i < \kappa\rho_2$), a seeker ($\kappa\rho_2 < \beta_i < \rho_2/\kappa$) or eager ($\beta_i > \rho_2/\kappa$) at h_2. The equilibria of G_2 and the induced continuation costs $(J_1(2), J_2(2))$ are as follows.

- One eager, the other eager or a seeker. They complete jointly on date 2, and $J_i(2) = \kappa c_2$.
- One eager (i), the other reluctant (j). Then $J_i(2) = c_2$ and $J_j(2) = c_3$.
- At least one reluctant and neither eager. They complete jointly on date 3, and $J_i(2) = \kappa c_3$.

- Both seekers. There are three equilibria, with $J_i(2) = \kappa c_2$, $J_i(2) = \kappa c_3$, or, in the mixed equilibrium in which j completes with probability $r_j = (c_2 - \kappa\beta_i c_3)/[(1-\kappa)(c_2 + \beta_i c_3)]$,

$$J_i(2) = r_i r_j \kappa c_2 + r_i(1-r_j)c_2 + (1-r_i)r_j c_3 + (1-r_i)(1-r_j)\kappa c_3.$$

Step 2 (solo continuations). $S_i(2) = c_2$ if $\beta_i > \rho_2$ and $S_i(2) = c_3$ if $\beta_i < \rho_2$.

Step 3 (date 1). For each continuation from Step 1, player i is willing to join if $\kappa c_1 < \beta_i S_i(2)$ and willing to go alone if $c_1 < \beta_i J_i(2)$. The equilibria of G_1 then follow from Proposition 2, with completion probabilities given by (3). A player who waits while her partner completes on date 1 completes on $\tau_i(2)$.

Combining the steps for every pair of motives yields the full correspondence. By Theorem 2(d), the equilibrium is unique and mixed exactly on $\mathcal{H}$ and its mirror image. (When the equilibrium is unique, G_2 has a unique, pure equilibrium, so mixing can occur only on date 1.) Multiple equilibria arise exactly when both players are seekers on date 2, or when both are seekers or both are avoiders on date 1 given a pure continuation.

C Replication files

The replication files implement the recursion in Lemma 2. The program `pgame.py` computes all equilibria of the game with simultaneous moves, including mixed equilibria, by solving the stage games backward, and `twomoment.py` does the same for the game with observable starts. The script `verify_paper.py` reproduces the numerical examples in the paper in exact rational arithmetic. For the introductory example, and for $c = (2, 3.5, 10)$, $\kappa = 0.7$ and $(\beta_1, \beta_2) = (0.18, 0.38)$, which satisfy the conditions of Corollary 2, it also checks every pure strategy profile of the full extensive form and confirms that none is an equilibrium. As a further check on Theorem 2(d), the script compares the characterization with the computed equilibria of randomly drawn three-period games, some drawn uniformly and some near $\mathcal{H}$, and finds no discrepancy. The program `make_figures.py` computes the shaded regions of Figure 1 and checks the curves in Figure 2 against the computed equilibria.