Optimal Taxation When Firms Can Underreport and Split

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Abstract

Firms can lower their taxes by underreporting their tax base and by splitting it among separately taxed entities. An authority chooses a tax schedule to raise a given revenue while minimizing firms' aggregate burden: tax payments plus manipulation costs. Firms differ in their bases and costs and choose sequences of underreports and splits. The optimal tax on each report equals the burden under the schedule that collects every reported unit, evaluated for a firm with that base and a cost cutoff chosen to meet the revenue requirement. I determine conditions under which the optimal tax under underreporting alone is unchanged when splitting is introduced. A tax designed for underreporting alone can lose almost all its intended revenue. Public registers of Slovak companies show both responses at the registration threshold of the value-added tax. Companies bunch below the threshold, and owners of unregistered companies are most likely to form a second company when the first approaches it.

1 Introduction

Firms can lower their taxes through real responses, legal avoidance or evasion (Slemrod, 1995; Slemrod and Yitzhaki, 2002; Saez et al., 2012). I treat all of these as costly manipulation of a firm's *potential tax base*, i.e. the base it would report without taxes or enforcement. I distinguish between two basic forms of manipulation: *underreporting*, which lowers the reported base, for example through avoidance expenditures or lower output, and *splitting*, which divides the base among separately taxed entities. Firms exhibit both responses (Sections 1.1 and 7), but models of optimal taxation with avoidance or evasion mostly focus on underreporting by treating each taxpayer as a single reporting unit (Section 1.2). I

characterize the optimal schedule in a model of taxation that allows firms to choose any sequence of both responses, show how it differs from a tax schedule designed with only underreporting in mind, and study the resulting fiscal consequences.

In the model, a tax authority commits to a report contingent schedule to raise a given revenue while minimizing firms' aggregate burden. The tax on a report can be neither negative nor larger than the report. Firms differ in their potential tax base and a cost parameter that converts manipulation effort into a real cost. Each chooses a costly *manipulation plan* consisting of a finite sequence of underreports and splits in order to minimize the sum of its tax payments and manipulation costs. This minimized sum is its *burden*.

There are two main results. First, the globally optimal tax schedule is characterized by a cost cutoff rule that maps a report r to a cost parameter $a(r)$. The tax on report r equals the burden of a firm with potential base r and cost parameter $a(r)$ under the full rate schedule, which is the schedule that collects the full reported amount (Theorem 4.2 and Corollary 4.3). Under the optimal schedule, a firm with potential base x and cost $c > a(x)$ reports truthfully through a single entity, while those with $c < a(x)$ retain their choices under the full rate schedule.

At the optimum, a slightly higher cutoff yields the same revenue per unit of added burden at every potential base. Raising the cutoff at one base raises the tax at that base, which is paid by the firms whose costs exceed the cutoff. It also leads the firms at the margin to manipulate, so that part of what they paid is now spent on manipulation. The fraction of the added burden that is lost in this way equals the generalized failure rate of the cost distribution at the cutoff, whatever efforts underreporting and splitting require. This rate is the percentage by which the share of firms above the cutoff falls when the cutoff rises by one percent. I assume that it increases with the cost parameter, so the fraction lost rises with the cutoff. The optimal schedule equalizes this fraction across bases, and the common value that meets the revenue requirement determines the marginal cost of public funds, the additional burden required to raise one more unit of revenue. If one third of the added burden is lost, for example, the marginal cost of public funds is 1.5 (Corollary 4.3).

A single schedule can implement these cutoffs at every base, even though a large firm can manipulate to a smaller report and pay the tax set for that report. That tax is what further manipulation from the smaller report would cost, under the full rate schedule, a firm whose cost is that report's cutoff. The large firm therefore gains nothing by this route as long as cutoffs do not rise with the base. I assume that, at any given cost, the generalized failure rate is no lower for larger firms, which ensures that they do not.

Second, I show how manipulation costs determine the role of splitting (Theorem 4.4). Plans with the same reports can require different effort, depending on the stage at which

the firm underreports (Figure 2). The curvature of manipulation costs determines whether underreporting before splitting, or splitting before underreporting, requires less effort. Under the theorem's concavity conditions, which hold for instance when larger entities underreport a unit of base with no more effort than smaller ones, a single entity can report any given total with no more effort than several. Dividing the base can then pay only if the schedule taxes the parts less than the whole. A subadditive schedule never does so, and no firm gains from splitting under any such schedule. The full rate schedule is additive, so firms' burdens under it are the same with and without splitting, and so is the optimal schedule.

The theorem's convexity conditions hold, for instance, when splitting costs depend only on the number of entities and smaller entities underreport a unit of base with no more effort than larger ones. Under these conditions, a firm that both splits and underreports does so with the least effort by dividing its base first and underreporting within the smaller entities. Facing the full rate schedule, a firm then does best either by reporting truthfully or by underreporting its entire base, with every entity reporting zero. A firm that underreports its entire base balances the saving from spreading its base across smaller entities against the cost of creating and reorganizing those entities. An optimal division uses equally sized entities. The tax equals the least effort needed for full underreporting times the cutoff, capped at the reported base.

A simple comparison identifies which firm sizes can benefit from splitting (Corollary 4.5). If dividing into two equal entities does not save enough on underreporting to cover the splitting cost, further division cannot help. The net gain from this division weakly increases with firm size, giving a size threshold.

Cheaper manipulation cannot benefit firms as a group once the schedule is redesigned to raise the same revenue (Proposition 4.6). Under a given schedule, access to splitting can only lower a firm's burden, but the redesigned schedule must raise the same revenue from firms that can manipulate more cheaply. Firms' opportunities to manipulate matter for the optimal schedule only through the burden that they bear under the full rate schedule. Access to splitting, a lower cost of creating entities, and any other change that leaves no firm with a higher burden under that schedule therefore act in the same direction. Revenue at the existing cutoffs cannot rise, so the authority must set cutoffs at least as high to meet the same target. The marginal cost of public funds then cannot be lower at any level of revenue that can still be raised, and firms' aggregate burden cannot fall. Firms below the size threshold can therefore face higher taxes even though splitting does not lower their effort for full underreporting.

Under a smooth and strict version of the convexity conditions of Theorem 4.4, the

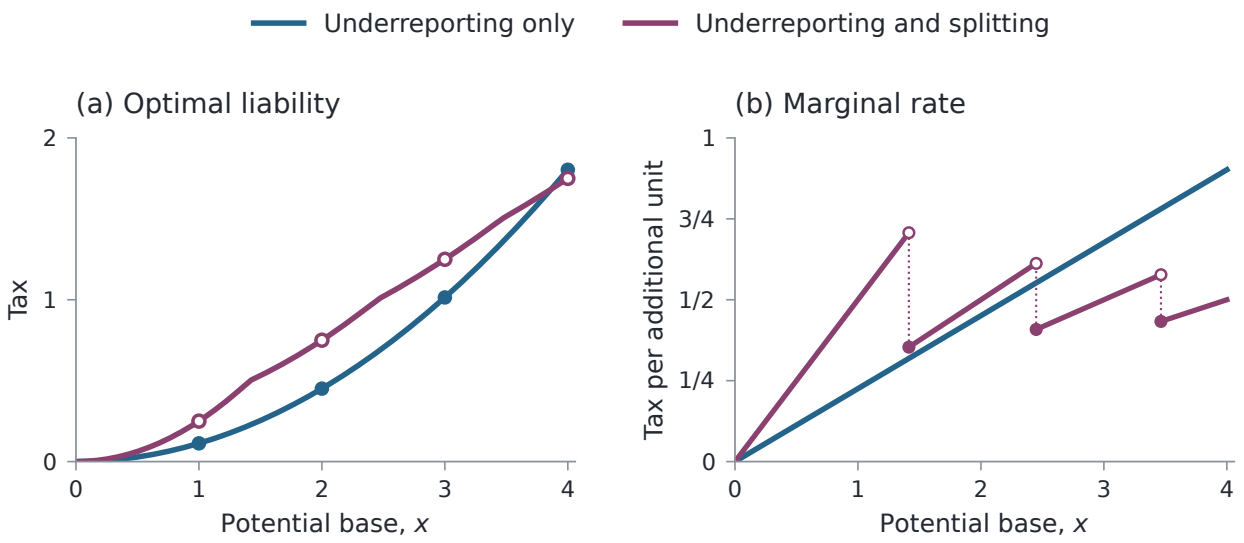


Figure 1: Splitting changes optimal liabilities and marginal rates. Potential bases are 1, 2, 3, and 4, each equally likely; costs are independent and uniform on $[0, 1]$. Underreporting z to r requires effort $z^2 - r^2$, and each additional entity requires effort 1. Both schedules raise $G = 3/4$. The cost cutoff with splitting is $1/4$. Curves show T_U^* and T_S^* on the full report domain.

discrete nature of splitting gives the optimal schedule a distinctive shape (Proposition 5.1). When the cost distribution is independent of the potential base and tax is below the reported base, marginal rates are nondecreasing while the optimal entity count remains fixed and fall when an additional entity becomes worthwhile. These downward changes in marginal rates do not require average tax rates to fall with firm size. If, in addition, splitting costs depend only on the number of entities, the optimal schedule lies below a proportional tax, capped at the reported base, by at most the cost of creating one entity at the cutoff cost. A restriction on the elasticity of underreporting effort then yields a single crossing, in which allowing splitting weakly raises optimal liabilities for firms below some size and weakly lowers them for larger firms (Proposition 5.2). Figure 1 illustrates these implications in the running example.

Ignoring splitting has a different consequence from redesigning taxes to accommodate it. At a common revenue requirement, the increase in firms' aggregate burden from access to splitting equals the additional manipulation cost that firms incur under the redesigned schedule. Under an optimal schedule, manipulation costs cannot exceed a multiple of revenue that is determined by the cost distribution and the cutoff, and this multiple is at most one half when costs are uniform (Proposition 6.1). In the example of Figure 1, manipulation costs under the optimal schedule rise from about 6 to about 17 percent of revenue when firms can split, and the marginal cost of public funds rises from about 1.15

to 1.5.

Keeping the schedule designed for underreporting alone after firms gain access to splitting can instead leave a large fiscal shortfall. At every firm size, expected tax payments are no higher than that schedule was designed to collect (Proposition 6.2). In the example of Figure 1 it collects only about two thirds of its target, although firms report the same total income as without splitting. In a quadratic-cost example with a single firm size, intended revenue grows in proportion to that size, while realized revenue grows only with its square root (Proposition 6.3). The realized share of intended revenue therefore converges to zero, even though a correctly designed schedule can still finance the original target. Firms can generate this shortfall largely by dividing reported income among entities while continuing to report it. Aggregate reported income alone can consequently miss the response that matters for revenue.

Slovak companies and their owners use both responses at the threshold above which a company must register for value-added tax (Section 7). The sales of companies bunch just below the threshold, and more strongly when the owner has two or more companies. Among sole owners of one unregistered company, the annual probability of forming a second company rises about tenfold as the sales of the first approach the threshold. It is then almost three times the probability for owners of registered companies of the same size, which varies little with sales. One year after forming a second company near the threshold, 30 percent of these owners report combined sales above the threshold through companies that are each below it and unregistered. A threshold in the corporate income tax at which far less is at stake moves the revenue that companies report, without a detectable change in the formation of companies. For a schedule with a threshold, the model predicts that firms slightly above it underreport and that larger firms split and report their whole base (Proposition 7.1).

The rest of the introduction reviews the empirical evidence and related literature. Section 2 sets out the model. Section 4 characterizes optimal taxes and firms' responses, and shows how taxes with finitely many brackets can approximate the optimum. Section 5 studies the shape of the schedule and the taxes faced by smaller and larger firms. Section 6 compares redesigning taxes with retaining a schedule that ignores splitting. Section 7 examines both responses at the Slovak registration threshold for value-added tax. Proofs appear in the appendix.

1.1 Empirical evidence

In the House Confecções case, an audit reviewed by Brazil's tax appeals council documented a clothing retail group that allocated sales among nominally separate companies to claim simplified tax treatment. The group also declared less than half of the revenue recorded in its own books (CARF, 2018). This case illustrates why the two responses need to be considered jointly. At the population level, Kuchumova et al. (2025) use a Russian threshold reform to document excess mass well below the original threshold and interpret the pattern through a model of underreporting and business splitting. Such responses can affect firms beyond the immediate neighborhood of a statutory threshold.

Organizational choices respond to both tax incentives and the costs of establishing entities. In the Netherlands, Massenz (2025) finds that a lasting tax advantage and the abolition of minimum capital requirements increased the number of companies per owner, without detectable changes in total assets, employment, or industry diversification across their companies. Japanese clustering below a value-added-tax threshold is also consistent with separate incorporation of business segments (Onji, 2009). Incorporation nevertheless entails more than a registration fee. Tazhitdinova (2020) documents requirements in the United Kingdom that include transferring assets and loans, establishing payroll, and maintaining corporate tax returns and accounts. These obligations motivate distinct costs of creating an entity and reorganizing the underlying business.

Section 7 adds evidence from Slovakia, where public registers link each company to its owners. Companies bunch below the registration threshold of the value-added tax, as they do in the United Kingdom and Finland (Liu et al., 2021; Harju et al., 2019), and the link to owners shows second companies being formed as the first approaches the threshold.

1.2 Related literature

The closest tax-design literature studies costly underreporting. Grochulski (2007) derives conditions under which an optimal nonlinear income tax induces no underreporting; Casamatta (2021, 2023) shows how other cost specifications permit avoidance at the optimum. Kopczuk (2001) studies heterogeneous avoidance opportunities, and Huang and Rios (2016) characterize nonlinear income taxes alongside an enforceable consumption tax. The present model adds an endogenous number of reporting units and derives the optimum from the composition of firms' manipulation plans. Its linear welfare objective and fixed potential bases differ from the redistributive and labor-supply environments in those papers. The treatment of manipulation as a real resource cost follows a distinction emphasized by Chetty (2009); payments that are transfers would require different welfare

accounting.

A second literature designs business-tax regimes around firm size and administrative costs (Keen and Mintz, 2004; Dharmapala et al., 2011; Kanbur and Keen, 2014; Sharma et al., 2025). Organizational responses complicate this design. Dabla-Norris et al. (2018) permit firm splitting through an exogenous probability of access and assume equal divisions using the fewest entities needed to avoid regulation. Kuchumova et al. (2025) study underreporting and splitting around a given tax threshold. Here the division, entity count, reporting decisions, and common tax schedule are chosen jointly. Equal-sized entities and the size threshold for splitting emerge from restrictions on manipulation costs. The distinction between redesign and deployment separates the distribution of liabilities at fixed revenue from the fiscal consequences of a fixed schedule.

Finally, creating reporting units connects taxation to multiple purchases in nonlinear pricing (Wilson, 1993; Alger, 1999), to splitting and merging in tax-allocation rules (Ju and Moreno-Ternero, 2011), and to the use of multiple identities in mechanism design (Yokoo et al., 2004). Subadditivity eliminates the direct saving from dividing a purchase or tax base. When firms can also underreport, a subadditive schedule still deters splitting under the concavity conditions of Theorem 4.4. Under its convexity conditions, firms can gain from splitting even when the schedule is additive, because dividing a firm can make later underreporting cheaper. In general, the comparison that deters splitting applies to the burden after optimal underreporting, adjusted for the costs of creating and restructuring entities, in place of the tax (Appendix C). This interaction also distinguishes the model from costly falsification with one reported characteristic (Lacker and Weinberg, 1989; Perez-Richet and Skreta, 2022, 2026). Optimal policy need not eliminate manipulation; it determines which cost types undertake it and how their organizational choices shape the tax schedule.

2 Model

A tax authority sets a common report contingent tax schedule, after which firms optimally manipulate their taxes in response to the schedule. Assumptions on the primitives that appear here are collected in Assumptions 2.1 below.

A unit mass of firms has potential tax bases $x \in [0, X]$, distributed according to μ, and cost parameters $c \in [0, 1]$, with conditional density f_x at each x. Here $0 < X < \infty$. An entity can **underreport**, reducing its base from z to $y \in [0, z]$, or **split**, dividing z among

$n \geq 2$ entities with bases $w_i \geq 0$ summing to z. The respective *efforts* are

$$U(z, y) = \phi(z) - \phi(y), \tag{1}$$

$$S(z; w_1, \ldots, w_n) = \kappa(n-1) + H(z) - \sum_{i=1}^{n} H(w_i). \tag{2}$$

A **manipulation plan** ω is a tree starting from x. Nonterminal nodes are the result of applying one of the two operations at a node and terminal nodes are reports. The effort at a nonterminal node is the effort of its operation (terminal nodes have zero effort) and total effort $K(\omega)$ of ω is the sum of effort across nodes. Let $r(\omega) = (r_1(\omega), \ldots, r_{N(\omega)}(\omega))$ be the reports of ω. The monetary cost of ω for a firm with cost parameter c is $cK(\omega)$. Let $\Omega_S(x)$ contain all such plans and $\Omega_U(x)$ those using only underreporting.

The authority chooses a common report contingent tax schedule, applied separately to each reporting entity, from

$$\mathcal{T} = \{T : [0, X] \to \mathbb{R}_+ : T \text{ is lower semicontinuous and } T(r) \leq r\}.$$

Under technology $j \in \{U, S\}$ and schedule T, each firm minimizes its total cost, the sum of tax payments and manipulation costs. Its **burden** is the resulting value:

$$V_T^j(x, c) = \min_{\omega \in \Omega_j(x)} \left\{ \sum_{i=1}^{N(\omega)} T(r_i(\omega)) + cK(\omega) \right\}.$$

For a revenue requirement $G \geq 0$, the authority solves

$$\min_{T \in \mathcal{T}} \mathbb{E}[V_T^j(x, c)] \qquad \text{subject to} \qquad \mathbb{E}[R_T^j(x, c)] \geq G, \tag{3}$$

where expectations use $\mu(\mathrm{d}x) f_x(c)\, \mathrm{d}c$. Equivalently, it maximizes aggregate firm surplus $W_j(T) = \mathbb{E}[x - V_T^j(x, c)]$. Denote optimal surplus by $W_j^*(G)$. Write $R_T^j(x, c)$ and $D_T^j(x, c)$ for tax payments and manipulation costs at a minimizing plan in response to a schedule T. A minimizing plan exists (Lemma A.3), and for almost every cost parameter all minimizing plans have the same tax payments and manipulation costs (Lemma A.4), so these expectations do not depend on how firms break ties.

2.1 Assumptions

The following assumptions on primitives are maintained throughout the paper.

Assumption 2.1 (Maintained Assumptions)**.**

(i) *Generalized failure rate and ordering.* The density $f_x(c)$ is positive on $(0,1)$. The generalized failure rate

$$h_x(c) = \frac{cf_x(c)}{1 - F_x(c)}$$

is jointly continuous on $[0, X] \times (0,1)$ and strictly increasing in c. For each $c \in (0,1)$, $h_x(c)$ is nondecreasing in x.

(ii) *Underreporting and splitting costs.* The fixed cost κ is strictly positive. The functions ϕ, H are continuous on $[0, X]$, with $\phi(0) = H(0) = 0$; ϕ is strictly increasing, and H is superadditive: $H(u+v) \geq H(u) + H(v)$ whenever $u, v \geq 0$ and $u + v \leq X$.

The forms (1) and (2) make each operation *path independent*. Consecutive underreporting steps require the same effort as one step from the starting base to the final report, and consecutive splitting steps require the same effort as one split of the starting entity into the final entities. Proposition A.2 shows that path independence, together with the continuity, symmetry, and positivity conditions stated there, is equivalent to these forms with κ, ϕ, and H as in Assumption 2.1(ii). A plan can therefore be taken to alternate between the two operations, and since each additional entity requires effort of at least κ, a firm never benefits from more than a bounded number of entities. These two facts give the existence of minimizing plans (Lemma A.3). Path independence does not extend across operations. Figure 2 shows two plans with identical reports whose efforts differ because they underreport at different stages.

The expression $h_x(c)$ in Assumption 2.1 (i) is the generalized failure rate (Lariviere, 2006). The quantity $1 - F_x(c)$ is the share of firms with base x whose costs exceed c. The expression $h_x(c)$ is the local percentage decline in this share per one-percent increase in c. The assumption that h_x is strictly increasing in c ensures that any optimal tax is μ almost everywhere equivalent to the optimal tax given by equation (15). The assumption that $h_x(c)$ is nondecreasing in x orders the cost distributions of small and large firms. It ensures that the optimal cost cutoff does not rise with firm size, which allows one schedule to implement the optimum at every base (Theorem 4.2). A useful special case is

$$F_x(c) = c^{\gamma(x)}, \qquad \gamma : [0, X] \to (0, \infty) \text{ continuous and nonincreasing.} \tag{4}$$

This will be used later to derive closed form expressions. Its generalized failure rate is $h_x(c) = \gamma(x)c^{\gamma(x)}/[1 - c^{\gamma(x)}]$, which increases with c and decreases with $\gamma(x)$, as verified in Appendix B. Thus a nonincreasing γ satisfies the required ordering. Constant γ makes costs independent of base; $\gamma = 1$ corresponds to uniform costs.

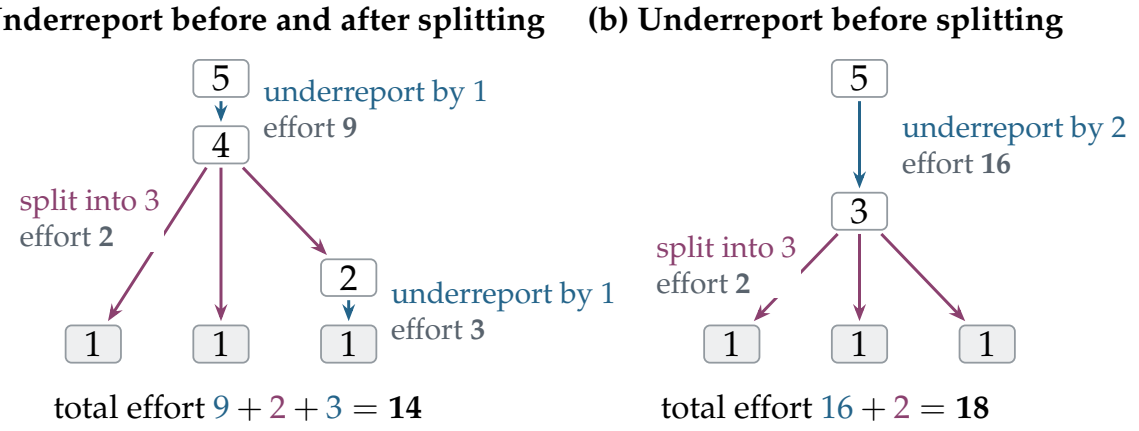


Figure 2: Identical total reports can require different effort. Each box is an entity; its label and vertical position give its base. Blue arrows reduce bases, and plum arrows form the children of a split. With $X = 5$, $\phi(z) = z^2$, $H = 0$, and $\kappa = 1$, both plans turn a base of 5 into three reports of 1 and pay $3T(1)$. Effort is 14 in (a) and 18 in (b), so manipulation costs are $14c$ and $18c$. Splitting into three entities requires effort 2 in either plan.

3 Examples

Monetary avoidance costs. In the monetary-cost formulation of Kopczuk (2001, p. 56), a taxpayer reduces taxable income by paying an avoidance cost. Hold earnings fixed at x, set transfers to zero, and specialize that cost to $c[\phi(x) - \phi(r)]$. Consumption is $C = x - T(r) - c[\phi(x) - \phi(r)]$, so any utility increasing in consumption gives the same cost-minimizing report. When avoidance uses real resources, the present welfare criterion measures aggregate consumption. This retains Kopczuk's monetary-cost mechanism; his endogenous labor supply and redistributive objective are additional features.

Labor supply. Hours worked provide the earnings response central to Mirrlees (1971). For a quasilinear labor-supply example, let the wage be 2, hours $q \in [0, 1]$, earnings $r = 2q$, and consumption $C = r - T(r)$. Give workers preferences

$$u(C, q) = C + 2(1 - q) - c(1 - q)^2.$$

Leisure is desirable and has diminishing marginal utility. With $x = X = 2$ and $\phi(z) = z - z^2/4$, these preferences satisfy

$$u = 2 - \{T(r) + c[\phi(2) - \phi(r)]\}.$$

Thus both individual choices and aggregate utility have exactly the paper's form. Here manipulation cost is forgone earnings net of the gain from leisure. Under $T(r) = tr$, a worker with $c > 0$ chooses $r = 2\max\{0, 1 - t/c\}$: $c = 1$ and $t = 1/2$ give earnings 1. Reporting the full potential base means retaining full-time hours; reduced earnings are still accurately reported.

4 Classification of optimal taxes

This section characterizes the tax schedule that raises a given revenue at minimum aggregate burden. A firm's burden rises with its cost parameter at a rate equal to its effort, which ties both expected revenue and expected burden to the effort that a schedule induces (Proposition 4.1). The optimal schedule taxes each report at the burden that a firm with that base and a cutoff cost would bear if every reported unit were collected (Theorem 4.2). The curvature of manipulation costs then determines the form of this tax and whether firms split (Theorem 4.4 and Corollary 4.5).

4.1 Burden, effort, and revenue

Fix an admissible schedule $T \in \mathcal{T}$ and a base x. A plan with tax payments τ and effort K costs a firm $\tau + cK$, which is linear in its cost parameter c. Burden is the lower envelope of these lines, so it is concave in c, and by the envelope theorem its slope is the effort of the plan the firm chooses (Lemma A.4 in Appendix A). For almost every c, every minimizing plan has effort $\partial_c V_T^j(x, c)$, and therefore

$$D_T^j(x, c) = c\,\partial_c V_T^j(x, c), \qquad R_T^j(x, c) = V_T^j(x, c) - c\,\partial_c V_T^j(x, c). \tag{5}$$

A firm with zero cost underreports its whole base for free, so $V_T^j(x, 0) = 0$ and burden is the integral of effort over lower costs,

$$V_T^j(x, c) = \int_0^c \partial_u V_T^j(x, u)\,\mathrm{d}u. \tag{6}$$

The reason is that a firm can copy the plan of a firm with the same base and a slightly lower cost, so the two burdens differ only by the extra cost of the effort in that plan. Write $\mathbb{E}[\,\cdot \mid x]$ for the expectation over cost parameters at base x.

Proposition 4.1 (Expected revenue and burden)**.** *Let Assumption 2.1 hold. For each technology $j \in \{U, S\}$, each schedule $T \in \mathcal{T}$, and each base x, expected tax payments and expected burden*

among the firms with base x are

$$\mathbb{E}[R_T^j \mid x] = \int_0^1 [1 - F_x(c) - cf_x(c)]\, \partial_c V_T^j(x, c)\, \mathrm{d}c, \tag{7}$$

$$\mathbb{E}[V_T^j \mid x] = \int_0^1 [1 - F_x(c)]\, \partial_c V_T^j(x, c)\, \mathrm{d}c. \tag{8}$$

Integrating over bases with μ gives expected revenue $\mathbb{E}R_T^j(x, c)$ and expected burden $\mathbb{E}V_T^j(x, c)$. The identities do not depend on which minimizing plans firms choose, and they do not use the monotonicity of the generalized failure rate.

Both identities are weighted sums of the effort of each cost type. By (6), effort by firms with cost c adds to the burden of every firm with the same base and a higher cost. These firms are a share $1 - F_x(c)$ of the firms at that base, which is the weight in (8). The effort itself is a resource cost of c per unit for a density $f_x(c)$ of firms, and nobody collects it. Subtracting it leaves the revenue weight $1 - F_x(c) - cf_x(c) = [1 - F_x(c)][1 - h_x(c)]$ in (7). The generalized failure rate $h_x(c)$ is thus the share of the burden created by effort at c that is dissipated in manipulation, and $1 - h_x(c)$ is the share collected as revenue. The identities are the counterparts for taxation of the virtual surplus formulas in screening problems (Myerson, 1981). The proof is in Appendix A.

4.2 The optimal tax

The building block is the burden under the full rate schedule $\mathrm{id}(r) = r$, which collects every reported unit. Write

$$C_j(a, x) = V_{\mathrm{id}}^j(x, a), \qquad j \in \{U, S\}, \tag{9}$$

for the burden of a firm with base x and cost parameter a under this schedule. The cost parameter is the first argument. Every admissible schedule satisfies $T \leq \mathrm{id}$, so the full rate burden is an upper bound on the burden under any schedule, $V_T^j(x, c) \leq C_j(c, x)$.

For $0 < \theta \leq 1$, define the cutoff $a_\theta(x)$ and the schedule $T_{j,\theta}$ by

$$h_x(a_\theta(x)) = \theta, \qquad T_{j,\theta}(r) = C_j(a_\theta(r), r). \tag{10}$$

Set $a_0(x) = 0$ and $T_{j,0} = 0$. Under Assumption 2.1(i), each generalized failure rate rises continuously from zero to infinity, so the cutoff exists and is unique. It is continuous, increasing in θ, and nonincreasing in the base x (Lemma B.1).

The schedule $T_{j,\theta}$ maximizes revenue net of $1-\theta$ times burden at every base, and it is therefore optimal for the revenue it raises.

Theorem 4.2 (Optimal tax schedule)**.** *Let Assumption 2.1 hold, and fix a technology $j \in \{U, S\}$ and a parameter $\theta \in [0,1]$.*

(i) *The schedule $T_{j,\theta}$ belongs to $\mathcal{T}$, and for every (x,c),*

$$V^j_{T_{j,\theta}}(x,c) = C_j\big(\min\{c, a_\theta(x)\}, x\big). \tag{11}$$

A firm with $0 < c < a_\theta(x)$ has the same minimizing plans as under the full rate schedule. A firm with $c > a_\theta(x)$ reports truthfully through one entity.

(ii) *For every $T \in \mathcal{T}$ and every base x,*

$$\mathbb{E}\big[R^j_T - (1-\theta)V^j_T \mid x\big] \le \mathbb{E}\big[R^j_{T_{j,\theta}} - (1-\theta)V^j_{T_{j,\theta}} \mid x\big], \tag{12}$$

with equality if and only if $V^j_T(x,c) = V^j_{T_{j,\theta}}(x,c)$ for every c.

(iii) *If $\mathbb{E}R^j_{T_{j,\theta}}(x,c) = G$, then $T_{j,\theta}$ solves the authority's problem (3). Every solution T raises exactly G and, for μ-almost every x, satisfies $T(x) = T_{j,\theta}(x)$ and $V^j_T(x,c) = V^j_{T_{j,\theta}}(x,c)$ for every c.*

The tax on report r equals the burden of a base-r firm under the full rate schedule, evaluated at $a_\theta(r)$. At each initial base, the cutoff firm can pay this amount directly and avoid manipulation. Higher-cost firms strictly prefer this payment to any positive-effort plan. Lower-cost firms retain their full rate choices, which may already involve truthful reporting. At the cutoff, truth is optimal and may tie with manipulation. Appendix B gives the proof.

Part (i) holds because a plan from x followed by a plan from each of its reports is again a plan from x, and because cutoffs do not rise with the base, so that every report $r \le x$ carries a cutoff $a_\theta(r) \ge a_\theta(x)$. A firm that manipulates down to a report r and then pays $T_{j,\theta}(r)$ pays exactly what further manipulation from r would cost a firm with the weakly higher cost $a_\theta(r)$ under the full rate schedule. Its total cost is therefore no lower than that of a full rate plan from x evaluated at the cost $\min\{c, a_\theta(x)\}$. If cutoffs rose with the base, a large firm could instead underreport to a size whose tax is calibrated to a lower cutoff than its own. The ordering of generalized failure rates in Assumption 2.1(i) rules this out.

The cutoff rule $h_x(a_\theta(x)) = \theta$ equalizes across bases the revenue obtained per unit of burden imposed. Raise the cutoff at base x from a to $a + \mathrm{d}a$. The tax at x rises by $K\,\mathrm{d}a$, where $K = \partial_a C_j(a,x)$ is the effort in the cutoff firm's full rate plan. The share $1 - F_x(a)$

of firms above the cutoff pays the increase, which adds $[1 - F_x(a)]K\,\mathrm{d}a$ to both revenue and burden. The mass $f_x(a)\,\mathrm{d}a$ of firms just above the old cutoff stops paying the tax and adopts the full rate plan. These firms are indifferent, so their burden does not change, but each of them now spends aK of what it used to pay on manipulation. Revenue therefore rises by $[1 - F_x(a) - af_x(a)]K\,\mathrm{d}a$, which is a fraction $1 - h_x(a)$ of the rise in burden. This fraction depends neither on the effort K nor on anything else about the manipulation technology. If it differed between two bases, shifting the cutoffs would raise the same revenue at a lower total burden. The optimum therefore sets $h_x(a(x))$ to a common value θ, and the technology enters the optimal tax only through the full rate burden C_j.

Part (ii) makes this marginal comparison global. By Proposition 4.1,

$$\mathbb{E}\left[R_T^j - (1-\theta)V_T^j \mid x\right] = \int_0^1 [1 - F_x(c)][\theta - h_x(c)]\,\partial_c V_T^j(x,c)\,\mathrm{d}c.$$

The weight on effort is positive and decreasing below $a_\theta(x)$ and negative above it. Integrating by parts below the cutoff turns the objective into a positive weight on the level of burden at every cost below the cutoff, less a penalty on effort above it. Burden can be no higher than the full rate burden, so the best profile follows the full rate burden up to the cutoff and is flat thereafter. This is the profile in (11).

Part (iii) is a Lagrangian argument in which $1/(1-\theta)$ is the multiplier on revenue. If $T_{j,\theta}$ raises G and $\theta < 1$, a feasible schedule with a lower expected burden would violate (12). At $\theta = 1$ the comparison bounds revenue alone, so every feasible schedule must attain the bound. The equality condition in part (ii) then fixes the burden of every firm at almost every base. The firms above the cutoff do not manipulate, so their common burden is the tax itself.

Matching the revenue requirement. Denote revenue and expected burden under $T_{j,\theta}$ by

$$Q_j(\theta) = \mathbb{E}R^j_{T_{j,\theta}}(x,c), \qquad L_j(\theta) = \mathbb{E}V^j_{T_{j,\theta}}(x,c).$$

Applying Proposition 4.1 to the burden in (11) gives

$$\begin{aligned} Q_j(\theta) &= \mathbb{E}_x \int_0^{a_\theta(x)} [1 - F_x(u)][1 - h_x(u)]\,\partial_u C_j(u,x)\,\mathrm{d}u, \\ L_j(\theta) &= \mathbb{E}_x \int_0^{a_\theta(x)} [1 - F_x(u)]\,\partial_u C_j(u,x)\,\mathrm{d}u. \end{aligned} \tag{13}$$

Here $\mathbb{E}_x$ averages over bases with distribution μ, and $\partial_u C_j(u,x)$ is effort under the full rate schedule. A higher parameter raises every cutoff. Between two parameters $\theta < \eta$, the

added effort comes from costs at which the generalized failure rate lies between θ and η, so each added unit of burden yields between $1-\eta$ and $1-\theta$ units of revenue.

Corollary 4.3 (Revenue matching and the marginal cost of public funds)**.** *Let the assumptions of Theorem 4.2 hold.*

(i) *The functions Q_j and L_j are continuous and nondecreasing on $[0,1]$, and for $0 \le \theta < \eta \le 1$,*

$$(1-\eta)[L_j(\eta) - L_j(\theta)] \le Q_j(\eta) - Q_j(\theta) \le (1-\theta)[L_j(\eta) - L_j(\theta)]. \tag{14}$$

Maximum attainable revenue is $G_{j,\max} = Q_j(1)$, and Q_j is strictly increasing until it reaches this value.

(ii) *For $0 \le G \le G_{j,\max}$, let $\theta_j(G)$ be the smallest parameter with $Q_j(\theta) = G$, and write $a_j(x) = a_{\theta_j(G)}(x)$. For $0 < G < G_{j,\max}$ this parameter is unique and lies in $(0,1)$. The schedule*

$$T_j^*(x;G) = C_j(a_j(x), x) \tag{15}$$

is optimal, with welfare $W_j^*(G) = \mathbb{E}x - L_j(\theta_j(G))$. Targets above $G_{j,\max}$ are infeasible.

(iii) *Optimal welfare W_j^* is concave, and for $0 < G < G_{j,\max}$ the marginal cost of public funds, which is the extra aggregate burden needed to raise one more unit of revenue, is*

$$-\frac{\mathrm{d}W_j^*(G)}{\mathrm{d}G} = \frac{1}{1-\theta_j(G)}. \tag{16}$$

The parameter θ is therefore the share of the marginal unit of burden that is spent on manipulation, and the marginal cost of public funds exceeds one by the manipulation cost per unit of revenue raised at the margin. It rises with the revenue requirement, and a higher target raises the cutoffs and weakly raises the scheduled liabilities $T_j^*(x;G)$. At $\theta = 1$ a higher cutoff no longer adds revenue.

The power-law cutoff. For the conditional power law in (4),

$$a_\theta(x) = \left[\frac{\theta}{\gamma(x)+\theta}\right]^{1/\gamma(x)}, \qquad a_1(x) = [1+\gamma(x)]^{-1/\gamma(x)}. \tag{17}$$

Whenever the cost distribution is independent of base, write its common optimal cutoff as $\alpha_j = a_j(x)$. Under the power law this holds when γ is constant. Uniform costs give $\alpha_j = \theta_j/(1+\theta_j) \le 1/2$. This case supplies the running example; the classification and the technology comparisons below hold for the full class in Assumption 2.1(i).

4.3 Classifying the optimal tax by cost curvature

The full rate burden in Theorem 4.2 has an explicit form for two classes of technologies. They are distinguished by the curvature of ϕ and of

$$\psi = \phi - H.$$

The function ψ determines the stage of a plan at which underreporting requires the least effort. Write the effort of a reduction from z to y as $[H(z) - H(y)] + [\psi(z) - \psi(y)]$. Every operation then contributes the value of H at its own base minus the values of H at the bases it produces, and these terms cancel along the tree. For a plan ω from x with reports $r_1, \dots, r_N$ and reductions from z_e to y_e,

$$K(\omega) = H(x) - \sum_{i=1}^{N} H(r_i) + \kappa(N-1) + \sum_e [\psi(z_e) - \psi(y_e)]. \tag{18}$$

Given its reports, a plan's effort depends on its tree only through the bases at which reductions take place. If ψ is convex, removing a given amount is cheapest at the smallest bases, so a firm splits first and then underreports within each entity. If ψ is concave, it is cheapest at the largest base, so a firm underreports first and then splits (Lemma C.1 in Appendix C). In Figure 2, $\psi(z) = z^2$ is convex and both plans reduce the base before the split. Splitting 5 into three equal entities and reducing each to 1 gives the same reports with effort $25/3 - 1$, about half the effort of plan (a).

When ψ is convex, the least effort needed to underreport the whole base is therefore

$$B(x) = H(x) + \min_{n \geq 1} \{ n\psi(x/n) + \kappa(n-1) \}. \tag{19}$$

For n equal entities, underreporting requires $n\phi(x/n)$ and splitting requires $\kappa(n-1) + H(x) - nH(x/n)$; their sum is the displayed objective. Convexity of ψ makes equal bases cheapest for each count n, and the creation charge bounds the optimal count.

Theorem 4.4 (Classification by cost curvature)**.** *Let the assumptions of Theorem 4.2 hold.*

(i) *If ϕ or ψ is concave, then $V_T^S = V_T^U$ for every schedule $T \in \mathcal{T}$ that is subadditive, meaning that $T(r + r') \leq T(r) + T(r')$ whenever $r + r' \leq X$. Under such a schedule, no minimizing plan of a firm with $c > 0$ uses more than one entity. In particular $C_S = C_U$, and the two technologies have the same T_j^*, $G_{j,\max}$, and $W_j^*(G)$.*

(ii) *If H and ψ are convex, then*

$$C_S(a, x) = \min\{x, aB(x)\}, \qquad C_U(a, x) = \min\{x, a\phi(x)\}. \tag{20}$$

In the convex case, Theorem 4.2 therefore gives

$$T_S^*(x) = \min\{x, a_S(x)B(x)\}, \qquad T_U^*(x) = \min\{x, a_U(x)\phi(x)\}. \tag{21}$$

In the first case, dividing a base never makes underreporting cheaper. When ψ is concave, firms underreport first, and a later split adds effort without changing the total reported. When ϕ is concave, the effort of underreporting one more unit does not rise with the size of the entity, so one entity reaches any total report with no more effort than several. Splitting could then help only by lowering the tax due on a given total report, which a subadditive schedule rules out. Each additional entity also requires effort of at least κ, so a firm with a positive cost parameter then uses a single entity. The full rate schedule is additive, so $C_S = C_U$.

In the second case, $\phi = H + \psi$ is convex and responses to the full rate schedule are all or nothing. An entity of base z facing the full rate schedule minimizes $r + a[\phi(z) - \phi(r)]$ over $0 \le r \le z$. This objective is concave in r, because the first unit an entity underreports requires the most effort. An endpoint, truthful reporting or full underreporting, therefore attains its minimum. The remaining issue is whether a firm should report some entities at zero and others at their full base. If such a mixture beats truth, its pieces reported at zero reveal a sufficiently cheap underreporting method. Convexity allows the firm to apply that method to the truthful residual and reduce its total cost further. Thus truthful reporting or full underreporting always attains the minimum. Appendix C proves this argument without relaxing the integer number of entities.

For $x > 0$, put $c_x = \min\{a_S(x), x/B(x)\}$, and set $c_0 = 0$. The ratio $x/B(x)$ is the cost at which full underreporting costs as much as paying the entire base; $a_S(x)$ is the policy cutoff. Almost every firm below their minimum reports zero through a division attaining $B(x)$; firms above it report truthfully. Thus costs determine whether to underreport, while technology and base determine how to divide. A firm *strictly prefers splitting* when every minimizing plan uses at least two entities.

Whether firms of a given size split depends on a comparison between one entity and two equal ones. This comparison requires only that ϕ and ψ be convex.

Corollary 4.5 (The size threshold for splitting)**.** *Let the assumptions of Theorem 4.2 hold, suppose ϕ and ψ are convex, and define*

$$d(x) = \psi(x) - 2\psi(x/2). \tag{22}$$

(i) *The function d is continuous and nondecreasing, and for every base x,*

$$C_S(a,x) < C_U(a,x) \text{ for some } a \iff B(x) < \phi(x) \iff d(x) > \kappa. \tag{23}$$

(ii) *Fix $0 < G \leq G_{S,\max}$. Under T_S^*, a positive share of the firms at base x strictly prefers splitting if and only if $d(x) > \kappa$, so the population share is positive if and only if $\mu\{x : d(x) > \kappa\} > 0$. If H is also convex, the share at base x is*

$$\mathbf{1}\{d(x) > \kappa\}\, F_x(c_x). \tag{24}$$

Under the conditional power law, $F_x(c_x) = c_x^{\gamma(x)}$. The test compares the underreporting saving from halving the base with the effort of that division:

$$\phi(x) - 2\phi(x/2) > \kappa + H(x) - 2H(x/2). \tag{25}$$

For convex ψ, the effort $n\psi(x/n) + \kappa(n-1)$ is convex in the count n (Boyd and Vandenberghe, 2004, Section 3.2.6). The saving from each additional entity therefore decreases: if the second does not pay for itself, later ones cannot. Convexity also makes $d(x)$ nondecreasing, so the sizes that benefit from splitting form an upper interval, possibly empty. Technology determines this size threshold; the revenue requirement determines how many cost types use splitting.

For the running example, let $\phi(x) = x^2$, $H = 0$, and $\kappa = 1$. Bases $1, 2, 3, 4$ are equally likely, and costs are independently uniform. At these bases, $n = x$ minimizes $x^2/n + n - 1$, giving $B(x) = 2x - 1$ and $\mathbb{E}B = 4$. At the relevant cutoffs the liability cap is slack: a share $1 - \alpha$ pays $\alpha B(x)$, giving revenue $\alpha(1-\alpha)\mathbb{E}B$. Thus $\alpha_S = 1/4$ raises $G = 3/4$. Replacing B by ϕ, whose mean is $15/2$, gives the cutoff $\alpha_U = (1 - \sqrt{3/5})/2$. With uniform costs the generalized failure rate is $h(c) = c/(1-c)$, so $\theta_S = \alpha_S/(1-\alpha_S) = 1/3$ and the marginal cost of public funds is $3/2$ with splitting, against about 1.15 without it.

Figure 3 shows the cutoff mechanism at base 2: low-cost firms spend resources to underreport, while high-cost firms pay the tax.

4.4 Fiscal and organizational consequences

A manipulation technology affects the optimal tax only through its full rate burden. Any change that lowers this burden, such as access to splitting or a lower cost of creating entities, therefore has the same qualitative effects. To state them, consider two technologies, each consisting of a plan set $j \in \{U, S\}$ and costs (κ, ϕ, H). Write C, Q, $G_{\max}$, $\theta(G)$, a, and W^*

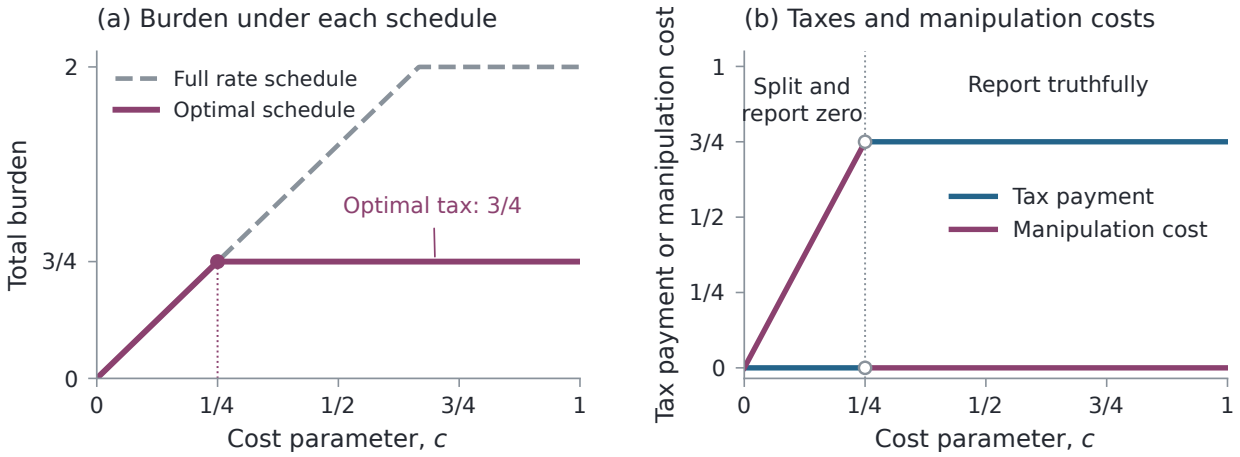


Figure 3: The cost cutoff separates underreporting from compliance. In the running example, a base of 2 can be fully underreported most cheaply by creating two entities of base 1, at total effort 3. Its burden under the full rate schedule is $\min\{2, 3c\}$. The optimal tax truncates this burden at $3/4$, giving the cutoff $c = 1/4$. Below it, the firm splits and underreports; above it, the firm reports truthfully. At the cutoff the two plans tie, which has probability zero.

for the objects C_j, Q_j, $G_{j,\max}$, $\theta_j(G)$, a_j, and W_j^* of the first technology, and mark those of the second with a tilde.

Proposition 4.6 (Cheaper manipulation)**.** *Let Assumption 2.1*(i) *hold and let both technologies satisfy Assumption 2.1*(ii)*. If $C \leq \widetilde{C}$, then $Q(\theta) \leq \widetilde{Q}(\theta)$ for every θ, so that $G_{\max} \leq \widetilde{G}_{\max}$, and for every $0 \leq G \leq G_{\max}$,*

$$\theta(G) \geq \widetilde{\theta}(G), \qquad a(x) \geq \widetilde{a}(x), \qquad W^*(G) \leq \widetilde{W}^*(G).$$

In particular, $G_{S,\max} \leq G_{U,\max}$ and, at every target feasible with splitting,

$$\theta_S(G) \geq \theta_U(G), \qquad a_S(x) \geq a_U(x), \qquad W_S^*(G) \leq W_U^*(G). \tag{26}$$

Holding other primitives fixed, a higher entity-creation effort κ weakly raises $G_{S,\max}$ and welfare at a common feasible target. It weakly lowers the optimal cutoffs and the share strictly preferring splitting. For almost every firm, every minimizing plan under the schedule in (15) *for the higher κ uses weakly fewer entities than every minimizing plan under the schedule for the lower κ.*

Cheaper manipulation therefore cannot make firms better off as a group when the authority must raise the same revenue. This holds even though access to splitting, or a lower cost of creating entities, weakly lowers every firm's burden under any given schedule. Integration by parts in (13) expresses revenue with nonnegative weights on full rate burdens, so lower burdens weakly reduce revenue at a given θ. If revenue falls,

restoring the target requires higher cutoffs. By (16) the marginal cost of public funds is then weakly higher at every level of revenue that the cheaper technology can raise, and aggregate burden is the integral of these marginal costs. Appendix B gives the proof.

Without curvature restrictions, Theorem 4.2 gives the population share strictly preferring splitting under T_S^*:

$$\int_{[0,X]} \int_0^{a_S(x)} \mathbf{1}\{C_S(c,x) < C_U(c,x)\}\, f_x(c)\, \mathrm{d}c\, \mu(\mathrm{d}x). \tag{27}$$

The share is nondecreasing in the revenue requirement: higher cutoffs allow more cost types to retain full rate choices, including those that strictly prefer splitting. Raising κ reduces splitting through two channels: it directly discourages additional entities and allows the authority to use lower cutoffs. Revealed preference orders entity counts even when optimal plans are not unique.

For the power law, the fraction undertaking any positive effort is at most

$$\mathbb{E}_x F_x(a_j(x)) = \mathbb{E}_x \frac{\theta_j(G)}{\gamma(x) + \theta_j(G)} \leq \mathbb{E}_x \frac{1}{1+\gamma(x)}.$$

Thus at most half the population manipulates under uniform costs. The bound concerns T_j^*; it allows firms below the cutoff to comply when their plan under the full rate schedule is already truthful.

4.5 Taxes with finitely many brackets

A finite bracket tax is continuous and linear between successive thresholds, as in Slemrod et al. (1994); Apps et al. (2014), and satisfies $0 \leq T(r) \leq r$. For $G < G_{j,\max}$, approximate T_j^* at a slightly higher revenue target, then scale its liabilities down to raise exactly G. Uniformly small changes in tax liabilities give small changes in aggregate revenue and welfare, even though individual firms may change plans abruptly (Lemma B.2). As bracket widths shrink, the welfare loss vanishes.

Corollary 4.7 (Approximation by finite bracket taxes)**.** *Maintain the assumptions of Theorem 4.2. Let $\mathcal{P}_k$ be finite partitions of $[0, X]$ whose largest interval tends to zero. Write $W_{j,k}(G)$ for optimal welfare among admissible continuous taxes linear on each interval of $\mathcal{P}_k$. For every $0 < G < G_{j,\max}$, an optimal schedule raising exactly G exists for all sufficiently large k, and*

$$W_{j,k}(G) \longrightarrow W_j^*(G).$$

If each partition retains all earlier thresholds, these values are nondecreasing wherever feasible.

Thresholds are fixed within each problem; marginal rates may be positive, zero, or negative. With costs independent of base and convex H, ψ, the approximation also works with nonnegative rates, by Proposition 5.1. At $G = G_{j,\max}$ there is no room to start from a higher target, and finite brackets may fail to raise the required revenue; Appendix B gives an example.

5 Tax shape and the distribution of liabilities

The classification separates the fiscal choice from the organizational choice. The revenue requirement selects the cutoff $a_S(x)$; the manipulation technology determines $B(x)$ and the number of entities used to attain it. Throughout this section, maintain the assumptions of Theorem 4.2, take H, ψ convex, and fix a positive target feasible with splitting. The comparisons concern scheduled liabilities $T_j^*(x)$; actual tax payments also depend on firms' manipulation choices.

5.1 Organizational thresholds and marginal rates

An entity of base z that is created and then fully underreported requires net effort $\psi(z) + \kappa$, a fixed creation charge plus a convex variable part. Its average per unit of base is $[\psi(z) + \kappa]/z$, and the least average effort is

$$\rho = \min_{0<z\leq X} \frac{\psi(z) + \kappa}{z}. \tag{28}$$

The minimum is attained at an efficient entity size, as in the textbook case of a fixed cost and a convex variable cost. A large firm can replicate entities of roughly that size, so its effort for full underreporting is close to $H(x) + \rho x$. Because the number of entities is an integer, the effort can differ from this amount, but it always lies between $H(x) + \rho x - \kappa$ and $H(x) + \rho x$. The lower bound subtracts the creation charge that the original entity does not pay.

Proposition 5.1 (The shape of the optimal tax)**.** *Let the assumptions of Theorem 4.2 hold, let H and ψ be convex, and fix a positive revenue requirement that is feasible with splitting.*

(i) *The function B is nondecreasing and satisfies*

$$H(x) + \rho x - \kappa \leq B(x) \leq H(x) + \rho x. \tag{29}$$

Hence

$$0 \leq \min\{x, a_S(x)[H(x) + \rho x]\} - T_S^*(x) \leq a_S(x)\kappa. \tag{30}$$

(ii) *If ψ is strictly convex, the optimal entity count for full underreporting is nondecreasing in the base, and at most two counts, which are adjacent, are optimal at any base. Counts n and $n+1$ are both optimal at x if and only if*

$$n\psi(x/n) - (n+1)\psi(x/(n+1)) = \kappa, \tag{31}$$

which holds for at most one base x_n.

(iii) *Suppose that ψ is strictly convex and that H, ψ, and a_S are continuously differentiable. Where count n is uniquely optimal and $a_S(x)B(x) < x$, so that the liability cap is slack, the marginal tax rate is*

$$T_S^{*\prime}(x) = a_S'(x)B(x) + a_S(x)[H'(x) + \psi'(x/n)]. \tag{32}$$

The marginal rate jumps downward at a threshold x_n with a slack cap. If the cost distribution is independent of base, liabilities are nondecreasing, and marginal rates are nondecreasing between consecutive thresholds where the liability cap is slack.

While the optimal entity count stays fixed, the marginal effort of fully underreporting a larger base is nondecreasing. At the next count threshold, an additional entity spreads the base more thinly and lowers marginal underreporting effort. Where the liability cap is slack, the optimal tax inherits both effects. When cost distributions vary with base, the declining cutoff adds $a_S'(x)B(x)$ to the rate; liabilities and rates within a branch need no longer increase. The downward rate change at an organizational threshold survives because the two entity counts require equal total effort at the threshold.

When H is linear, restructuring costs cancel from each division. With a common cost distribution, the comparison schedule in (30) is proportional, subject to the liability cap. The optimal tax lies within $\alpha_S\kappa$ of this schedule; the gap becomes small relative to the base as firm size grows.

In the running example, $B(x) = \min_n\{x^2/n + n - 1\}$, $\rho = 2$, and $2x - 1 \leq B(x) \leq 2x$. Count thresholds are $\sqrt{n(n+1)}$; between them the marginal rate is $x/(2n)$. The marginal rate falls at each threshold, while the average rate $T_S^*(x)/x$ never falls, because $B(x)/x$ is nondecreasing on each branch. At each populated integer base $x \in \{1, 2, 3, 4\}$, $n = x$ attains $B(x) = 2x - 1$. Thus one common underreporting technology generates the effort bound, the integer organizational choices, and the changing marginal rates in Figure 1. Figure 4 shows how the effort envelope and the entity count fit together.

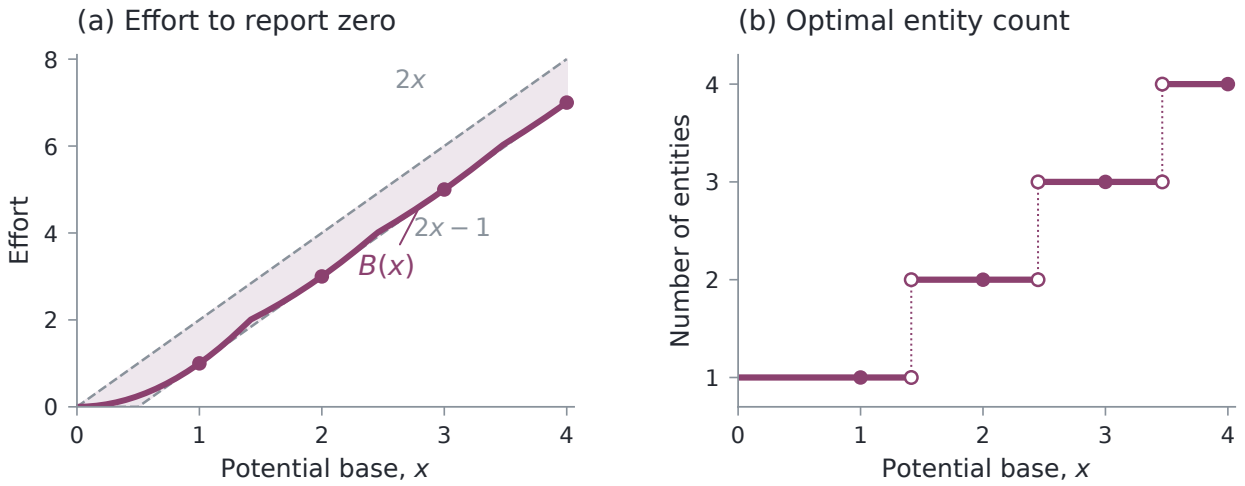


Figure 4: One technology determines the effort envelope and the entity count. The running example has $\phi(x) = x^2$, $H = 0$, and $\kappa = 1$. The least effort for full underreporting lies between $2x - 1$ and $2x$. At each integer base, using entities of base 1 attains the lower bound. Between integers, the optimal count changes where adjacent effort branches meet. Points mark populated bases; both adjacent counts are optimal at a threshold.

5.2 Which firms finance the adjustment?

Firms too small to benefit from splitting under the full rate schedule can face higher optimal liabilities when it becomes available. If $d(x) \leq \kappa$, Corollary 4.5 gives $C_S(a, x) = C_U(a, x)$. The cutoff ordering in (26) therefore implies

$$T_S^*(x) \geq T_U^*(x). \tag{33}$$

Their burdens rise weakly at every cost parameter as well. Splitting does not lower their full rate burden, but financing the same revenue target requires weakly higher cutoffs.

An elasticity restriction extends this comparison to the whole schedule. The ratio $x\phi'(x)/\phi(x)$ measures the percentage increase in full-underreporting effort in one entity following a one-percent increase in base.

Proposition 5.2 (A single crossing of optimal taxes)**.** *Suppose the cost distribution is independent of base, H is linear, and ϕ is convex and continuously differentiable on $(0, X]$. If $x\phi'(x)/\phi(x)$ is nondecreasing, there is $x_c \in [0, X]$ such that*

$$T_S^*(x) \geq T_U^*(x) \quad (x < x_c), \qquad T_S^*(x) \leq T_U^*(x) \quad (x > x_c). \tag{34}$$

Equality regions are allowed.

The elasticity condition makes $B(x)/\phi(x)$ nonincreasing: larger firms obtain weakly

greater proportional underreporting savings from splitting. By (21), uncapped liabilities have ratio $(\alpha_S/\alpha_U)[B(x)/\phi(x)]$, so this ratio can cross one only from above. The liability cap preserves the ordering. The result uses an elasticity restriction in addition to convexity; with varying cost distributions, the cutoff ratio also varies with base.

For the running example, the elasticity equals 2. The splitting tax is $(2x-1)/4$ at the four populated bases. At the same revenue $G = 3/4$, the underreporting tax is $\alpha_U x^2$, where $\alpha_U = (1-\sqrt{3/5})/2$. The first three sizes pay higher scheduled liabilities after redesign, while the largest pays less. The example illustrates a redistribution of liabilities toward firms with less scope to reduce underreporting costs through division.

6 The cost of ignoring splitting

Splitting changes both the cost of financing public spending and the revenue from a given schedule. The first comparison below allows the authority to redesign its tax at a common revenue requirement. The second applies its original tax after firms gain access to splitting. Throughout this section, maintain Assumption 2.1.

6.1 Redesign at a common revenue requirement

Write $D_j^* = D^j_{T_j^*}(x,c)$ and $R_j^* = R^j_{T_j^*}(x,c)$ for manipulation cost and tax payments under the optimal tax. Since revenue equals G,

$$W_j^*(G) = \mathbb{E}x - G - \mathbb{E}D_j^*, \qquad j \in \{U,S\}.$$

The welfare loss from splitting is therefore the additional manipulation cost under the redesigned tax. Under any optimal tax, manipulation costs are at most a multiple of revenue that depends only on the cost distribution and the cutoff. For a cutoff $a \in (0,1)$ at base x, define

$$\lambda_x(a) = \frac{\int_0^a c f_x(c)\,\mathrm{d}c}{a[1-F_x(a)]}. \tag{35}$$

The numerator is the cost of one unit of effort by every firm below the cutoff, and the denominator is the revenue from a tax of a on every firm above it. Their ratio is the manipulation cost per unit of revenue when all firms below the cutoff exert the same effort. When the cost distribution is independent of base, write λ for λ_x.

Proposition 6.1 (Manipulation costs and the welfare loss from splitting)**.** *Let the assumptions of Theorem 4.2 hold, and fix a positive revenue requirement G feasible with splitting.*

(i) *For $j \in \{U, S\}$ and every base x,*

$$\mathbb{E}[D_j^* \mid x] \leq \lambda_x(a_j(x))\, \mathbb{E}[R_j^* \mid x], \tag{36}$$

with equality if the full rate burden $C_j(\cdot, x)$ is linear on $[0, a_j(x)]$.

(ii) *The welfare loss from splitting satisfies*

$$0 \leq W_U^*(G) - W_S^*(G) = \mathbb{E}D_S^* - \mathbb{E}D_U^* \leq G \sup_x \lambda_x(a_S(x)).$$

(iii) *Under the conditional power law in (4), $\lambda_x(a_\theta(x)) = \theta/(\gamma(x)+1)$. With $\gamma_{\min} = \min_x \gamma(x) > 0$, therefore, $\mathbb{E}D_j^* \leq \theta_j(G)G/(\gamma_{\min}+1)$ for $j \in \{U, S\}$, and*

$$0 \leq W_U^*(G) - W_S^*(G) \leq \frac{\theta_S(G)}{\gamma_{\min}+1} G \leq \frac{G}{\gamma_{\min}+1}. \tag{37}$$

For every constant $\gamma > 0$ and fixed $\theta \in (0, 1]$, there are economies with $\theta_S(G) = \theta$ whose welfare loss divided by G approaches $\theta/(\gamma+1)$.

Concavity of burden in the cost parameter is the key restriction. Firms with lower costs exert weakly more effort, and effort at low costs yields the most revenue per unit of manipulation cost. Manipulation costs are therefore largest relative to revenue when effort is the same at all costs below the cutoff, as in the all-or-nothing response of Theorem 4.4(ii) with a slack liability cap. Then (36) holds with equality. If in addition the cost distribution is independent of base, manipulation costs are exactly $\lambda(\alpha_j)G$ under either technology, and the welfare loss from splitting is $[\lambda(\alpha_S) - \lambda(\alpha_U)]G$. Splitting then lowers welfare only through the higher cutoff it requires. Within a given economy, a lower target requires a lower $\theta_S(G)$ and tightens the bound. Appendix D proves the inequality and constructs economies approaching it.

With uniform costs, $\lambda(a_\theta) = \theta/2$ and the universal bound is half the revenue being raised. In the running example both taxes raise $3/4$. Manipulation costs are $1/8$ with splitting, which equals $\theta_S G/2$, and about 0.048 without it, or about 17 and 6 percent of revenue. The welfare comparison keeps the provision of public revenue fixed.

6.2 Applying the original tax after splitting becomes possible

Fix $0 < G \leq G_{U,\max}$. Let $P^U(x, c)$ be the burden anticipated under the no-splitting optimum $T_U^*(r) = C_U(a_U(r), r)$, and let $P^M(x, c)$ be the burden realized when firms can split under that same schedule. Write R^U and R^M for the corresponding anticipated and

realized tax payments. Define

$$\delta_x(c) = P^U(x,c) - P^M(x,c), \qquad m_x = \mathbb{E}[\delta_x(c) \mid x], \qquad e_x = \delta_x(1).$$

Here m_x is mean burden relief among firms at base x, and e_x is the relief of its highest-cost firms. Let $m = \mathbb{E}_x m_x$, and let Q^M, D^M denote realized revenue and mean manipulation costs. Write $D^U = \mathbb{E}D_U^*$ for anticipated mean manipulation costs.

Under the conditional power law in (4), integrating (5) over costs gives, for every admissible schedule,

$$\mathbb{E}[R_T^j \mid x] = (1+\gamma(x))\mathbb{E}[V_T^j \mid x] - \gamma(x)V_T^j(x,1). \tag{38}$$

Conditional revenue thus depends on mean burden and the burden of the highest-cost firm.

Proposition 6.2 (The fiscal effect of an unanticipated splitting option)**.** *Let the assumptions of Theorem 4.2 hold and fix* $0 < G \leq G_{U,\max}$.

(i) *The realized burden satisfies*

$$\begin{aligned} P^M(x,c) &= C_S(c,x), && c \leq a_U(x), \\ C_S(a_U(x),x) \leq P^M(x,c) &\leq C_U(a_U(x),x), && c \geq a_U(x). \end{aligned} \tag{39}$$

(ii) *At every base, expected tax payments fall by at least* $1-\theta_U(G)$ *times the mean burden relief,*

$$\mathbb{E}[R^U - R^M \mid x] \geq [1-\theta_U(G)]\, m_x \geq 0. \tag{40}$$

Hence, if $G < G_{U,\max}$*,*

$$\frac{G - Q^M}{1-\theta_U(G)} \geq m. \tag{41}$$

(iii) *Under the conditional power law, the exact decomposition is*

$$G - Q^M = \mathbb{E}_x[(\gamma(x)+1)m_x - \gamma(x)e_x] \geq 0, \tag{42}$$

$$D^M - D^U = \mathbb{E}_x[\gamma(x)(m_x - e_x)]. \tag{43}$$

Part (ii) follows from Theorem 4.2(ii) in one step. When firms can split, no schedule attains a higher value of revenue less $1-\theta_U(G)$ times burden, at any base, than the schedule $T_{S,\theta}$ with $\theta = \theta_U(G)$. That value is no higher than its counterpart without splitting, which the anticipated outcome attains. By (16), inequality (41) values lost revenue

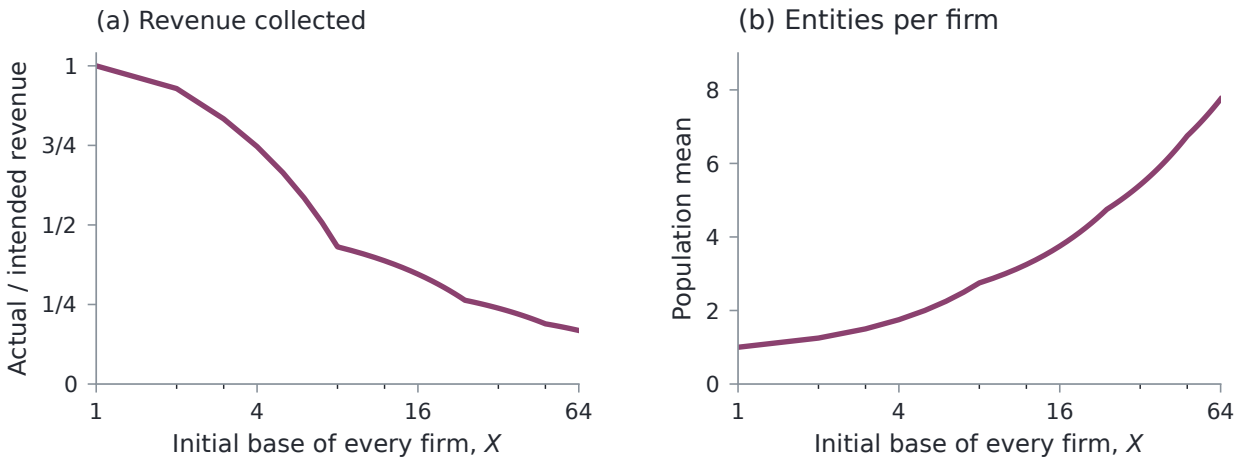


Figure 5: A feasible target can be missed by almost its entire amount. All firms have integer base X, costs are uniform, and $\phi(r) = r^2$, $H = 0$, $\kappa = 1$. The authority applies $T_U^*(r) = r^2/(4X)$, intended to raise $G_X = (X/4)(1 - 1/(4X))$. Firms can split and underreport in any finite sequence. Actual revenue is computed by integrating tax payments over the exact cost intervals for each optimal entity count. The correctly designed tax can raise $(2X - 1)/4$, which exceeds the target at every plotted size. The horizontal axes use a logarithmic scale.

at the original marginal cost of public funds; that value covers firms' gain. Subtracting (38) for anticipated and realized burdens gives the decomposition in part (iii). If the highest-cost firms still comply, $e_x = 0$: one dollar of mean burden relief at base x costs the authority $1 + \gamma(x)$ dollars and adds $\gamma(x)$ dollars to manipulation costs. If they also manipulate, the endpoint correction matters.

In the running example, all highest-cost firms comply, so $e_x = 0$. The original schedule collects about half a unit when the target is three quarters. Total reported income is the same as without splitting, because firms above the original cutoff still report their entire base. The fiscal shortfall is twice firms' burden relief, and additional manipulation costs equal that relief. Appendix D.4 gives exact integration formulas for this response. Section 7 documents owners who divide a business at a registration threshold and continue to report its sales.

6.3 Almost complete loss of intended revenue

The shortfall can be arbitrarily large as a fraction of the intended revenue, even when the target remains feasible after redesign (Figure 5).

Proposition 6.3 (Severe fiscal failure)**.** *Let $\phi(r) = r^2$, $H = 0$, $\kappa = 1$, and let costs be uniform on $[0, 1]$. All firms have initial base X, a positive integer. Fix $0 < b < 1/2$. The no-splitting optimum*

$$T_U^*(r) = \frac{b}{X} r^2 \tag{44}$$

raises $G_X = bX(1 - b/X) = b(X - b)$, which is less than the maximum revenue $(2X - 1)/4$ with splitting. If firms can split under this schedule, realized revenue Q_X^M and mean manipulation costs D_X^M satisfy

$$\left| Q_X^M - \tfrac{2}{3}\sqrt{bX} \right| \le 2, \qquad \left| D_X^M - \tfrac{2}{3}\sqrt{bX} \right| \le 2. \tag{45}$$

Hence $Q_X^M / G_X \to 0$ as $X \to \infty$, at the rate $X^{-1/2}$.

Firms with $c > b/X$ report the entire base but divide it equally among a number of entities minimizing

$$\frac{bX}{n} + c(n - 1), \qquad n = 1, 2, \ldots. \tag{46}$$

The first term is the tax payment; the second is the cost of additional entities. Ignoring integer rounding, the minimizing count is $\max\{1, \sqrt{bX/c}\}$. Intended revenue grows in proportion to X, whereas actual revenue grows only in proportion to $\sqrt{X}$. Lower-cost firms underreport their entire base. The example exposes the fiscal importance of organizational responses even when firms above the original cutoff continue to report their full total base.

7 Evidence from a registration threshold

Slovak companies and their owners respond to a threshold in the value-added tax by underreporting and by splitting. The sales of companies bunch just below the threshold. Owners of companies that have not registered for the tax become more likely to form a second company as the sales of the first approach the threshold, and some owners keep two companies just below it. A much smaller jump in the corporate income tax moves the revenue that companies report, without a detectable effect on the formation of companies. Proposition 7.1 derives these responses from the model for a schedule with a threshold.

7.1 The threshold

A company that is registered for the value-added tax (VAT) charges the tax on its sales, at a standard rate of 20 percent in the years studied here, and deducts the tax included in its purchases. An unregistered company does neither. A company must register once its sales in any period of at most twelve consecutive months reach EUR 49,790, and it may

register voluntarily below this threshold.[1] A company that sells to registered companies loses little by registering, because its customers deduct the tax that it charges. Many small companies of this kind register voluntarily, as Liu et al. (2021) document for the United Kingdom. A company that sells to households cannot pass the tax on in this way. At a rate of 20 percent the tax is one sixth of the price that a customer pays. If prices stay unchanged, a registered company therefore gives up one sixth of its sales and recovers one sixth of its taxed purchases. A company with sales of EUR 60,000 and purchases of EUR 24,000 would lose EUR 6,000 a year, one tenth of its sales, in addition to the cost of filing returns (Harju et al., 2019). The loss applies to all sales as soon as the threshold is reached, so the liability jumps at the threshold. Such a jump is called a *notch* (Kleven and Waseem, 2013).

The threshold applies to each company separately. Most incorporated small businesses in Slovakia are limited liability companies, which a single person can form and own. A person who divides a business between two such companies owes no VAT as long as each company stays below the threshold. Dividing a business is lawful, but the tax administration can disregard an arrangement whose main purpose is a tax advantage, which it has to establish.[2] A second company must be entered in the commercial register and file its own annual accounts, and the customers and staff of the business must be assigned to one company or the other. These requirements correspond to the model's efforts of creating entities and of reorganizing the base among them. Until 2026 the Commercial Code also allowed a person to be the sole owner of at most three limited liability companies.[3]

7.2 Predicted responses

In the model's terms, the business of one owner, meaning the whole of the owner's activity, is a firm. Its potential base x is the sales that it would report if there were no threshold, and each company is an entity. The threshold corresponds to the schedule

$$T_b(r) = \tau r \, \mathbf{1}\{r > b\},$$

[1] Act No. 222/2004 Coll. on Value Added Tax, Section 4. The threshold was unchanged from 2016 to 2023, the years used below. The law requires registration when sales reach the threshold, and the schedule T_b below taxes reports strictly above it. The difference matters only for a company whose sales equal the threshold exactly.

[2] Tax Procedure Code, Act No. 563/2009 Coll., Section 3(6).

[3] Commercial Code, Act No. 513/1991 Coll., Section 105a, repealed by Act No. 29/2026 Coll. with effect from 17 August 2026.

where b is the threshold and $\tau \in (0, 1]$ is the loss from registration per unit of sales. In the example above $\tau = 1/10$. The loss differs across businesses, and the results below apply to each business with its own τ. The schedule belongs to $\mathcal{T}$, and the model describes how firms respond to any schedule in $\mathcal{T}$, whether or not it is optimal, because a firm's plan minimizes the sum of its taxes and manipulation costs.

A firm whose base exceeds the threshold has three options. It can report its base and pay τx, it can underreport down to the threshold, or it can divide its base between two entities that both stay at or below the threshold. For bases up to twice the threshold, the firm's burden is the cost of the cheapest of these options. I show this for the simplest technology that satisfies the convexity conditions of Theorem 4.4, in which the underreporting effort ϕ is convex and splitting costs depend only on the number of entities, so that $H = 0$.

Proposition 7.1 (Responses to a threshold)**.** *Let Assumption 2.1*(ii) *hold with $H = 0$ and ϕ convex, and let $2b \leq X$. Under the schedule T_b, a firm with base $x \in (b, 2b]$ and cost parameter $c > 0$ has burden*

$$V^S_{T_b}(x, c) = \min\{\tau x,\ c[\phi(x) - \phi(b)],\ c\kappa\}.$$

If the first term is the unique minimum, the firm reports x through one entity. If the second term is the unique minimum, the firm reports b through one entity. If the third term is the unique minimum, the firm uses two entities, whose reports are at most b and sum to x.

The proof is in Appendix E. A second entity requires effort κ whatever the base, while underreporting the excess over the threshold requires little effort when the excess is small. Suppose that a second entity requires less effort than underreporting a base of $2b$ down to the threshold, so that $\phi(2b) - \phi(b) > \kappa$, and let $\hat{x} \in (b, 2b)$ solve $\phi(\hat{x}) - \phi(b) = \kappa$. A firm with a base between b and $\hat{x}$ that manipulates reports exactly b through one entity. A firm with a base between $\hat{x}$ and $2b$ that manipulates divides into two entities and reports its whole base. Whether a firm underreports or splits therefore depends only on its size, and its cost parameter determines only whether it manipulates at all. Corollary 4.5 obtains its size threshold from the same comparison of one entity with two, applied there to the full rate schedule. With the technology of the running example, $\phi(x) = x^2$ and $\kappa = 1$, and a threshold at $b = 2$, firms with bases between 2 and $\hat{x} = \sqrt{5} \approx 2.24$ that manipulate report exactly 2, and firms with bases between $\sqrt{5}$ and 4 that manipulate split.

Entities therefore bunch at the threshold. For bases up to $2b$, no firm that manipulates has an entity above b, and a firm with a base between b and $\hat{x}$ that manipulates reports exactly b. A business for which registration is not costly has no reason to manipulate.

A firm that manipulates splits once its base exceeds $\hat{x}$. Its entities then report at most b each, and their reports sum to its base, which exceeds b. With $H = 0$ every division

of the base into two parts of at most b requires the same effort, so the proposition does not determine how such a firm divides its base. If reorganizing the base is costly and H is strictly convex, the division that requires the least effort is the most unequal one, in which one entity keeps a base of b and the other receives the remainder $x - b$. A firm whose base is slightly above $2b$ can keep two entities at or below the threshold only by underreporting. Under the supposition above and if $X > 2b$, such a firm does best, when it manipulates, with two entities that both report exactly b. Appendix E proves both statements. Some owners should therefore have one company at the threshold and a smaller second company, and some should have two companies at the threshold.

The two responses depend differently on the size of the notch. A firm with a base x between $\hat{x}$ and $2b$ splits if and only if $c < \tau x/\kappa$. At any such base the share of firms that split is therefore at most $F_x(\min\{1, 2\tau b/\kappa\})$, which shrinks to zero with τ. Underreporting is different. However small τ is, every firm whose base is close enough to the threshold that $\phi(x) - \phi(b) < \min\{\tau x, \kappa\}$ underreports to the threshold, whatever its cost parameter. A small notch therefore empties a narrow band of reports just above the threshold, while few firms split.

7.3 Data

The evidence combines three public registers. The commercial register records the shareholders of every company and the dates on which they hold their shares. The Register of Financial Statements holds the annual accounts that every company must file. The tax administration publishes the list of companies that are registered for VAT. The registers identify a shareholder by name and address. I treat shareholder records with the same full name and address as one person, whom I call an owner, and I follow an owner through a change of address that is recorded within the shareholder history of one company. An owner is the *sole owner* of a company if the register records no other shareholder. Businesses that are divided among relatives or associates are not linked in this way.

I use two samples, which Appendix E describes in detail. The first is drawn from the 108,196 owners who were the sole owner of exactly one limited liability company at the end of 2016, and covers the years 2017 to 2023. Accounts have to be retrieved company by company, so I use a sample. It contains all 3,165 of these owners who became a shareholder of a newly formed company in those years, and a random sample of 2,887 of the other 105,031. The latter are weighted by the inverse of their sampling probability. The second sample is a cross-section of owners at the end of 2019, with the accounts of their companies for 2019 and 2020. It contains all 9,507 sole owners of two or more companies and a random

sample of 20,000 of the 129,664 sole owners of one company incorporated before 2019. Accounts that cover the calendar year 2019 are available for 28,376 of the 40,874 companies in the cross-section. For most of the others the register contains no accounts for that year.

Sales are the sum of sales of goods and sales of own products and services in the income statement, the items closest to the sales that count toward the threshold. A company that stays unregistered must keep its sales below the threshold in every period of twelve months, and so in every calendar year. The absence of unregistered companies above the threshold therefore shows only that the law is applied. The evidence is in the distribution of all companies, registered or not, and in how close to the threshold the unregistered ones are. Sales of services that are exempt from VAT, as in health care, education, finance, and the letting of real estate, do not count toward the threshold. Companies in industries with such services are one tenth of the cross-section. They are included in the results below, and excluding them changes little (Appendix E).

7.4 Companies near the threshold

The sales of companies bunch just below the threshold, which means that many more companies report sales just below it than just above it (Figure 6). Bunching of this kind is known from other value-added taxes (Onji, 2009; Harju et al., 2019; Liu et al., 2021). Among the sampled owners of one company, the band of about EUR 2,500 just below the threshold contains 272 companies, compared with an average of 219 in the three bands before it and 145 in the band just above it. In the model, bunching is the response of firms whose bases exceed the threshold by less than $\hat{x} - b$. The accounts cannot show whether these companies sold less or reported less. Either is a costly reduction of the base, which is how the model treats underreporting.

Registered companies show where the bunching comes from. Below the threshold registration is voluntary, and 43 percent of the companies in (a) are registered. These are the companies for which registration is not costly, and they do not bunch. The band just below the threshold contains 95 of them, against an average of 107 in the three bands before it. Above the threshold every company must register, so the number of registered companies would jump there if companies did not avoid crossing it. It is 109 in the band just above the threshold.

Bunching is stronger among companies whose owner has two or more companies. In (b) the band just below the threshold contains 410 companies. This is 1.5 times the average of 270 in the three bands before it, against a ratio of 1.2 in (a), and four times the 103 in the band just above it. A division in which one company of a business stays at the threshold,

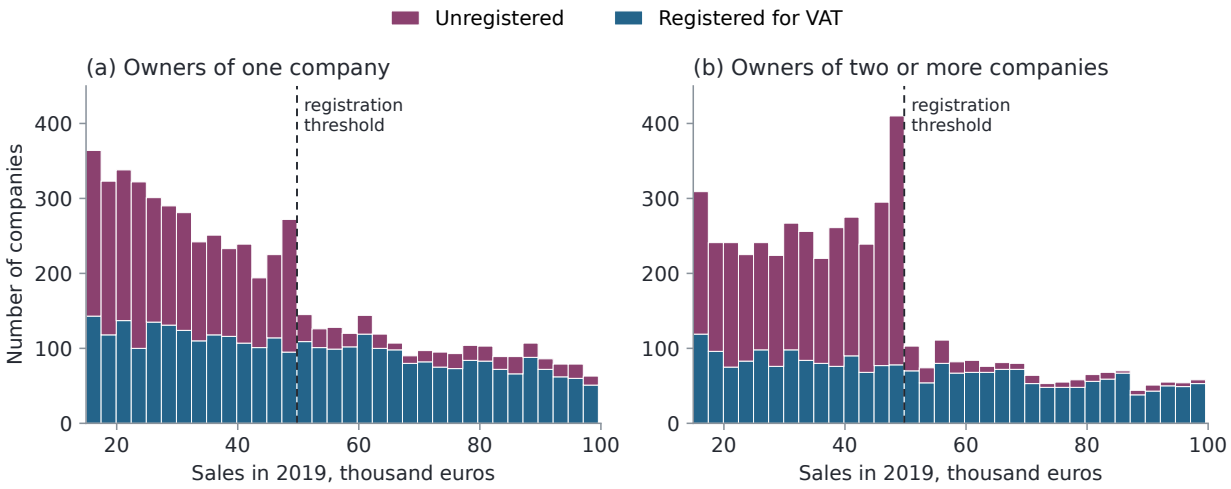


Figure 6: The sales of companies bunch just below the registration threshold, and more so when the owner has two or more companies. Bars count companies in bands of 5 percent of the threshold of EUR 49,790, about EUR 2,500. Sales are sales of goods and services in the accounts for 2019, and registration is measured at the end of 2019. Panel (a) contains the 5,938 companies with sales between 30 and 200 percent of the threshold in the random sample of sole owners of one company. Panel (b) contains the 5,090 such companies of all sole owners of two or more companies. Unregistered companies above the threshold include those with exempt sales (Appendix E).

as costly reorganization implies, produces this pattern.

Some owners of two companies keep both just below the threshold (Figure 7). Of the 9,507 owners of two or more companies, 7,194 had exactly two, both incorporated before 2019, and accounts for both companies in 2019 are available for 4,602 of them. The others have a company without accounts for that year. For 77 of the 4,602 owners, both companies have sales within 20 percent below the threshold. If the companies of the 4,602 owners were paired at random, 30 owners would be expected to show this pattern, and none of 5,000 random pairings produces more than 48. Random pairing preserves the sales of every company, including any bunching of its own, so the excess reflects the joint position of the two companies of one owner. The companies of one owner are of similar size more often than chance at lower sales as well, but less markedly. For windows of the same relative width that end at EUR 35,000 and at EUR 40,000, the observed number of owners is 1.8 and 1.4 times the random number, against 2.6 times at the threshold.

The 77 owners also differ from others in what their companies do. Where the industry of both companies is recorded, at the two-digit level of the standard classification, the two companies are in the same industry in 49 percent of these cases, against 24 percent for other owners of two companies with positive sales. In 62 of the 77 cases neither company is registered. An owner who keeps two unrelated businesses just below the threshold

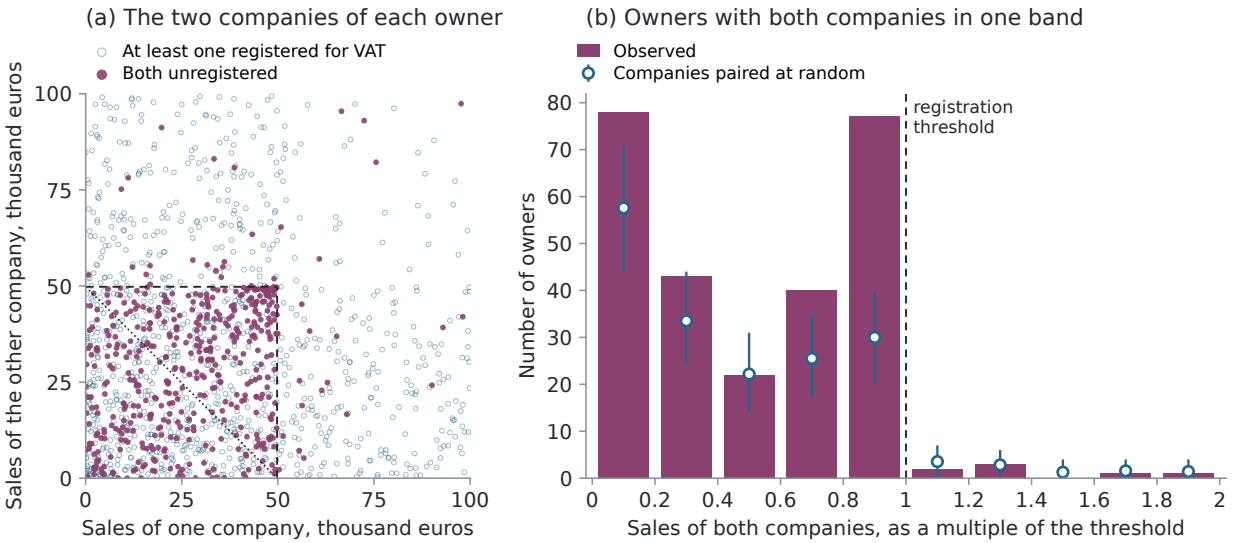


Figure 7: More owners of two companies have both just below the threshold than random pairing would produce. The owners are the 4,602 sole owners of exactly two companies at the end of 2019 with accounts for both. In (a), each point is one owner, placed at the sales of the two companies in 2019, for the 1,488 owners whose companies both have positive sales of at most EUR 100,000. Dashed lines mark the threshold for each company, and combined sales exceed the threshold above the dotted line. In (b), bars count the owners whose two companies fall in the same band of sales, in bands of one fifth of the threshold. Circles show the mean and the central 95 percent of the same counts when the companies are paired at random 5,000 times.

would also appear among the 77, but the shared industry suggests that many of them run one business through two companies. Their reports are those that the model predicts for a firm whose base is close to twice the threshold. Among 550 owners of three companies, 32 have every company below the threshold and combined sales above twice the threshold, against 10 when their companies are regrouped at random.

The pattern is weaker in 2020. Both companies are near the threshold for 57 owners, 2.2 times the random number, but the factor is almost as large for the window that ends at EUR 40,000, so the accounts for 2020 do not single out the threshold. In that year sales fell for 60 percent of the companies that had sales between half the threshold and the threshold in 2019. Of the 69 owners with both companies near the threshold in 2019 and accounts for 2020, 24 still had both there.

7.5 The formation of second companies

Owners of unregistered companies become more likely to form a second company as the sales of the first approach the threshold (Figure 8). The potential base of a business is

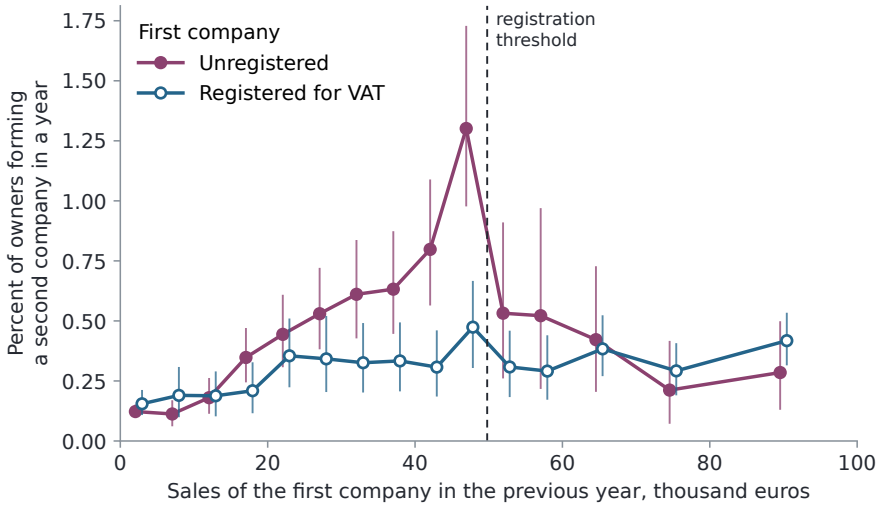


Figure 8: Owners of unregistered companies become more likely to form a second company as the first approaches the registration threshold. Points are the annual percentage of sole owners of one company who form a second company as its sole owner, by the sales of the first company in the previous year and its VAT registration at the end of that year. The owners are those who were the sole owner of exactly one company at the end of 2016, observed from 2017 to 2023 until they first hold another company. The band just below the threshold runs from EUR 45,000 to EUR 49,790. Vertical lines are 95 percent confidence intervals. Unregistered companies with sales above the threshold are few, and 62 percent of them are in industries with exempt services.

not observed. A business that has outgrown the threshold while its company remains unregistered reports sales just below it, so this is where the model places the firms that split. Among sole owners of one unregistered company, the annual probability of forming a second company, again as its sole owner, is 0.13 percent when the first company had positive sales of at most EUR 10,000 in the previous year. It rises steadily with those sales and reaches 1.3 percent when they were between EUR 45,000 and the threshold, ten times as much, although still a rare event. For owners of registered companies the probability lies between 0.3 and 0.5 percent at all sales from EUR 20,000 to EUR 100,000.

The difference between unregistered and registered companies grows as sales approach the threshold. Within 20 percent below the threshold it is 0.67 percentage points, with a 95 percent confidence interval from 0.42 to 0.95. In the last band, from EUR 45,000 to the threshold, it is 0.83 points, with an interval from 0.45 to 1.29. Between the last two bands the probability rises by 0.50 points for unregistered companies, with an interval from 0.09 to 0.94, and by 0.17 points for registered ones. The difference between these two increases has an interval from -0.13 to 0.85, so the data do not establish that the increase in the last EUR 5,000 is confined to unregistered companies.

Two other differences between the groups could produce such a gap. Unregistered

Table 1: Owners one and two years after a second company is formed

	Second company formed	
	Yes	No
One year later		
First company's sales above the threshold	18	21
Owner's combined sales above the threshold	68	21
Combined sales above the threshold, no company registered	36	11
At least one company registered for VAT	44	16
Median sales of the first company, thousand euros	40	43
Observations	126	675
Two years later		
First company's sales above the threshold	22	22
Owner's combined sales above the threshold	71	22
Combined sales above the threshold, no company registered	29	11
At least one company registered for VAT	54	19
Median sales of the first company, thousand euros	35	41
Observations	108	605

Notes. The sample is sole owners of one company that, in a given year, was unregistered and had sales within 20 percent below the threshold. The first column contains the owners who formed a second company as its sole owner in the next year, and the second column the owner-years in which no second company was formed, weighted by the inverse of the sampling probability. Outcomes are measured one and two years after the year in which the second company was or could have been formed, for owners whose accounts are available. Entries are percentages of owners in the first column and of owner-years in the second, except for medians and observations. In the second column, combined sales are the sales of the one company.

companies near the threshold may be growing faster, and owners of growing businesses form new companies. They are also two and a half years younger on average, and owners of young companies form new ones more often. Neither accounts for the gap. Within 20 percent below the threshold, it is about 0.5 points both for companies whose sales grew by more than 10 percent in the preceding year and for the others. The probability is also higher for unregistered than for registered companies in each age group (Appendix E). The lower income tax rate described below cannot account for it either, because that rate did not exist before 2020.

The gap is largest at the start of the panel. Within 20 percent below the threshold it is 1.2 points in 2017 and 2018, with an interval from 0.75 to 1.66, and 0.4 points from 2019 to 2023, with an interval from 0.11 to 0.67. The panel follows owners who had one company at the end of 2016, and an owner who forms a second company leaves it. The owners who remain in later years are therefore those who had not formed one earlier, and I cannot tell this selection apart from a change in behavior.

What follows the formation of a second company is mixed (Table 1). Take the 151

owners who formed a second company when the first was unregistered and had sales within 20 percent below the threshold. Accounts for both companies one year later are available for 126 of them. Their combined sales then exceed the threshold for 68 percent, and for 36 percent they do so while neither company is registered. Among comparable owners who did not form a second company, 11 percent have sales above the threshold and remain unregistered. For 30 percent of the owners who formed a second company, the reports are those that Proposition 7.1 predicts for a firm that splits, with each company at or below the threshold, combined sales above it, and no company registered. Formation is also followed by registration more often. A year later 44 percent of these owners have a registered company, against 16 percent of the others. A second company is therefore a way around registration for about a third of the owners who form one near the threshold, and part of an expansion that ends in registration for many of the others.

The first company's sales fall by more than those of comparable companies when a second company is formed. Its median sales are EUR 46,000 in the year before and EUR 40,000 in the year after, while the median for comparable owners falls from EUR 45,000 to EUR 43,000. The new company has median sales of EUR 25,000, so the owner's additional sales appear in the new company. The new company is in the same industry as the first in 30 percent of cases.

7.6 A smaller notch

The corporate income tax contains a second notch, at which much less is at stake. In 2020 a company whose revenue did not exceed EUR 100,000 paid 15 percent of its profit in tax instead of 21 percent. From 2021 to 2023 the ceiling for the lower rate was the VAT registration threshold of EUR 49,790.[4] I measure a company's revenue by the sum of operating and financial revenue in its accounts, which is close to the taxable revenue to which the law refers. The lower rate applies to the whole profit, so a company just above the ceiling pays 6 percent of its profit more than a company just below it. For a company with revenue of EUR 100,000 and a profit of EUR 10,000 the difference is EUR 600 a year, one tenth of the loss from VAT registration in the example above.

The revenue that companies report bunches below this ceiling. In 2020, 212 companies in the cross-section report revenue in the EUR 2,500 below the ceiling and 84 in the EUR 2,500 above it. In 2019, before the lower rate existed, the same bands contain 138 and 132 companies. The accounts also record the lower rate. In 2020 the median ratio of tax to

[4]Income Tax Act, Act No. 595/2003 Coll., Section 15, as amended by Acts No. 315/2019 Coll. and No. 416/2020 Coll.

profit is 15 percent for companies with revenue between EUR 80,000 and the ceiling and 21 percent for companies with revenue up to EUR 20,000 above it.

The formation of companies shows no detectable response. An owner whose company had revenue between EUR 100,000 and EUR 200,000 could bring all of it under the ceiling in 2020 by dividing it between two companies. Among owners of registered companies, which are not subject to the VAT notch, the annual probability of forming a second company was 0.51 percent for such owners in 2017 to 2019 and 0.29 percent in 2020. For owners whose company had revenue between EUR 49,790 and EUR 100,000, who had no such reason in 2020, it fell by the same amount, from 0.48 to 0.26 percent. The difference between the two changes is zero, with a 95 percent confidence interval from -0.23 to 0.23 percentage points, which excludes an increase half as large as the probability before 2020. From 2021 to 2023 the lower ceiling gave owners with revenue between EUR 49,790 and EUR 100,000 the same reason to divide, and their probability did not rise relative to that of owners with revenue above EUR 100,000 either. The corresponding difference is -0.16 points, with an interval from -0.35 to 0.03. Owners who already have two companies do not place both just below the ceiling. In 2020, both companies have revenue within 20 percent below the ceiling for 7 owners, against 6 under random pairing.

The contrast between the two notches is what Proposition 7.1 leads one to expect. Every firm that is close enough to a notch underreports, so even a small notch produces bunching. A second company requires the same effort whatever the notch, and a saving of a few hundred euros a year rarely covers it.

7.7 Interpretation

The evidence concerns how firms respond to a given schedule, which is the part of the model on which the characterization of optimal taxes rests. Both responses appear at one threshold. Companies bunch below it, and owners of unregistered companies form second companies more often as the first approaches it. About a third of those who do so near the threshold then report combined sales above the threshold while no company of theirs is registered. The size at which a business splits cannot be observed, because its potential base is not, so the comparison between underreporting and splitting in Proposition 7.1 is not tested directly.

Owners who divide a business in this way keep reporting its sales. In the 2019 cross-section, 266 owners of two or three companies outside the industries with exempt services have every company below the threshold and unregistered while their combined sales exceed it. Together these owners report sales of about EUR 20 million, or EUR 75,000

per owner, a small amount for the economy as a whole. Some of them run unrelated companies, so the figure is not a measure of tax lost, and it leaves out businesses that are divided among relatives or partners, which the registers do not link. It illustrates the mechanism of Section 6, in which a schedule that ignores splitting can lose revenue although reported sales do not fall.

The effort that an additional entity requires also depends on policy. The repeal of the limit of three companies per sole owner in 2026 lets owners at the limit form further companies. In the model, changes that make splitting cheaper cannot benefit firms as a group once the schedule is redesigned to raise the same revenue (Proposition 4.6).

The accounts measure reported sales, so they cannot separate lower sales from unreported sales among the companies that bunch. An owner can also form a second company near the threshold for reasons unrelated to VAT, although such reasons would not make the probability rise with sales for unregistered companies alone.

8 Conclusion

When businesses choose their reporting units, tax design must account for the cost of both underreporting a base and dividing it. The optimal schedule is pinned down by the burden firms can attain under the full rate schedule and by a cost cutoff that matches revenue. This characterization makes the reporting unit part of the tax problem while retaining a tractable global solution.

The economic implications depend on which comparison policy makes. Redesign shifts some of the financing burden toward firms whose manipulation opportunities change least. Holding the original schedule fixed instead can undermine revenue even when total reported income barely changes. At the Slovak registration threshold, some owners divide growing businesses among companies that each stay below the threshold while reporting combined sales above it. Organizational thresholds and payments aggregated across commonly controlled entities are therefore central objects for evaluating a tax system. The results characterize optimal taxes under separate assessment; studying consolidation, endogenous enforcement, and production alongside organizational choice requires a richer instrument and welfare environment.

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Appendix

A Foundations

A.1 Effort accounting and path independence

Lemma A.1 (Effort accounting)**.** *Let Assumption 2.1*(ii) *hold. Every plan $\omega \in \Omega_S(x)$ with reports $r_1, \dots, r_N$ and reductions from z_e to y_e satisfies* (18) *and $K(\omega) \geq \kappa(N-1)$. Merging two consecutive operations of the same kind into one changes neither the reports nor the effort. A plan in $\Omega_U(x)$ with report r has effort $\phi(x) - \phi(r)$.*

Proof. Write the effort of a reduction from z to y as $[H(z) - H(y)] + [\psi(z) - \psi(y)]$. Every operation then contributes H at its own base minus the sum of H over the bases it produces, and over the tree these terms telescope to $H(x) - \sum_i H(r_i)$. Splits add κ for every child beyond the first, in total $\kappa(N-1)$, and reductions add their differences in ψ. This is (18). A split into n children requires effort of at least $\kappa(n-1)$ by superadditivity of H, so $K(\omega) \geq \kappa(N-1)$. Merging consecutive reductions leaves the sum of the differences in ψ unchanged, and merging consecutive splits changes no term of (18). Without splits, $N = 1$ and the reductions merge into one from x to r. □

Without splitting, the firm's problem is therefore

$$V_T^U(x, c) = c\phi(x) + \min_{0 \leq r \leq x} \{T(r) - c\phi(r)\}. \tag{47}$$

The next proposition shows that the potential forms of the two efforts are equivalent to path independence within each operation.

Proposition A.2 (Potential representations and alternating operations)**.** *The following conditions on operation costs are equivalent to the representations* (1)–(2) *with κ, ϕ, H satisfying Assumption 2.1*(ii)*:*

(i) *The function U is continuous on $0 \leq y \leq z \leq X$, with $U(z, z) = 0$ and $U(z, y) > 0$ for $y < z$. For $0 \leq u \leq y \leq z \leq X$,*

$$U(z, u) = U(z, y) + U(y, u). \tag{48}$$

(ii) *For each number of children, S is continuous in their bases and symmetric in their labels.*

For $n, m \geq 2$, $\sum_i w_i = z \leq X$, and $\sum_{k=1}^{m} v_k = w_n$, with all bases nonnegative,

$$S(z; w_1, \ldots, w_{n-1}, v_1, \ldots, v_m) = S(z; w_1, \ldots, w_n) + S(w_n; v_1, \ldots, v_m). \tag{49}$$

Every binary split satisfies $S(u+v; u, v) \geq S(0; 0, 0) > 0$ whenever $u, v \geq 0$ and $u + v \leq X$.

The normalized reduction potential ϕ is unique; the normalized restructuring potential H is unique up to a linear term. The creation charge is $\kappa = S(0; 0, 0) = S(x; x, 0)$. Every finite manipulation tree can be replaced by an alternating tree, in which no operation is immediately followed by another of the same kind, with the same terminal reports and total effort.

Proof. The representations imply the two conditions because differences of ϕ and of H telescope, ϕ is strictly increasing, H is superadditive, and $\kappa > 0$. The last statement is the merging of operations in Lemma A.1, which removes a node each time it is applied. Conversely, suppose the two conditions hold. Setting $u = 0$ in (48) gives (1) with $\phi(z) = U(z, 0)$. This function is continuous, vanishes at zero, and is strictly increasing because $U(z, y) > 0$ for $y < z$, and it is the only potential with $\phi(0) = 0$ that represents U. Let $\kappa = S(0; 0, 0)$ and $q(u, v) = S(u + v; u, v) - \kappa$. Symmetry and (49), applied to the two groupings of three children, give the cocycle equation

$$q(u, v) + q(u + v, w) = q(v, w) + q(u, v + w), \qquad u + v + w \leq X.$$

Setting $v = w = 0$ gives $q(u, 0) = 0$, so that $S(x; x, 0) = \kappa$. A continuous symmetric solution of the cocycle equation on this triangle has the form $q(u, v) = H(u + v) - H(u) - H(v)$ for a continuous H with $H(0) = 0$.[5] Since $q \geq 0$, H is superadditive. Induction on the number of children, using (49), gives (2). Two such potentials differ by a continuous additive function, which is linear. □

A.2 Existence and the envelope

Lemma A.3 (Existence of minimizing plans)**.** *Let Assumption 2.1*(ii) *hold. For every $T \in \mathcal{T}$, $j \in \{U, S\}$, and (x, c), the minimum defining $V_T^j(x, c)$ is attained. The burden V_T^j is lower*

[5] See Ng (1974), as stated in Ebanks (1981, p. 254). A continuous representative is

$$H(x) = \frac{4}{X}\left[\int_0^{x/2} q(t, X/2)\, \mathrm{d}t - \int_0^{X/2} q(x/2, t)\, \mathrm{d}t\right] + q(x/2, x/2),$$

as substitution of the cocycle equation verifies.

semicontinuous on $[0, X] \times [0, 1]$*, and it is continuous if* T *is continuous. Every minimizing plan at* $c > 0$ *satisfies*

$$K \leq \phi(x), \qquad N \leq 1 + \left\lfloor \frac{\phi(x)}{\kappa} \right\rfloor \leq M = 1 + \left\lfloor \frac{\phi(X)}{\kappa} \right\rfloor. \tag{50}$$

Proof. Since $0 \leq T(0) \leq 0$, reporting zero through one entity costs $c\phi(x)$. At $c > 0$, a plan with effort above $\phi(x)$ is therefore strictly dominated, and so is a plan with more than $1 + \lfloor \phi(x)/\kappa \rfloor$ reports, since its effort is at least $\kappa(N-1)$ by Lemma A.1. At $c = 0$, reporting zero through one entity attains the minimum, which is zero. It thus suffices to minimize over plans with at most M reports in which no reduction follows another (Lemma A.1). Such a plan has at most M reports and fewer than M splits, and each of its reductions is followed by a split or a report, so it has fewer than $4M$ nodes. Only finitely many tree shapes therefore arise. Fix one such shape. The bases at its nodes are the vectors $x\beta$, where $\beta \geq 0$ ranges over a compact polytope that does not depend on x. In this polytope the root has $\beta = 1$, the children of a split sum to their parent, and a reduced base is at most its parent. Because ϕ and H are continuous and T is lower semicontinuous, the cost of the plan is lower semicontinuous in (β, x, c), and it is continuous if T is. Its minimum over β is attained by Weierstrass's theorem. This minimum is lower semicontinuous in (x, c), and continuous if T is (Aliprantis and Border, 2006, Lemma 17.30 and Theorem 17.31). The minimum over the finitely many shapes inherits these properties. □

Lemma A.4 (Burden and effort)**.** *Let Assumption 2.1*(ii) *hold, fix* $T \in \mathcal{T}$*,* $j \in \{U, S\}$*, and* x*, and write* $V(c) = V_T^j(x, c)$*. The function* V *is concave, nondecreasing, and* $\phi(x)$*-Lipschitz on* $[0, 1]$*, with* $V(0) = 0$ *and* $V(c) \leq V_{\mathrm{id}}^j(x, c) \leq x$*. At every* $c \in (0, 1)$ *at which* V *is differentiable, hence at almost every* c*, every minimizing plan has effort* $V'(c)$ *and tax payments* $V(c) - cV'(c)$*.*

Proof. By Lemma A.3, V is the minimum of the affine functions $c \mapsto \sum_i T(r_i(\omega)) + cK(\omega)$ over the plans with effort in $[0, \phi(x)]$. It is therefore concave, nondecreasing, and $\phi(x)$-Lipschitz, and $V(0) = 0$ because $T(0) = 0$. The bound $T \leq \mathrm{id}$ gives $V \leq V_{\mathrm{id}}^j(x, \cdot)$, and truthful reporting gives $V_{\mathrm{id}}^j(x, c) \leq x$. The last statement is the envelope theorem (Milgrom and Segal, 2002, Theorem 1). □

Proof of Proposition 4.1. Fix x and write $V(c) = V_T^j(x, c)$. By Lemma A.4, V is Lipschitz with $V(0) = 0$, so (6) holds, and (5) holds for almost every c at every minimizing plan. Expected revenue and burden therefore do not depend on how ties are broken. Tonelli's theorem gives

$$\mathbb{E}[V_T^j \mid x] = \int_0^1 \int_0^c V'(u)\, \mathrm{d}u\, f_x(c)\, \mathrm{d}c = \int_0^1 [1 - F_x(u)]\, V'(u)\, \mathrm{d}u,$$

$$\mathbb{E}[D_T^j \mid x] = \int_0^1 c f_x(c)\, V'(c)\, \mathrm{d}c.$$

The first identity is (8), and subtracting the second from it gives (7), since $R_T^j = V_T^j - D_T^j$. For the aggregate statement, V_T^j is lower semicontinuous, hence Borel measurable, and its derivative in c is a limit of difference quotients, so the integrands are jointly measurable. They are bounded by $\phi(X)[1 - F_x(c) + c f_x(c)]$. □

B Optimality and its consequences

Lemma B.1 (Cutoffs)**.** *Let Assumption 2.1*(i) *hold. For every x, the generalized failure rate h_x maps $(0,1)$ onto $(0,\infty)$ and extends continuously to $h_x(0) = 0$. For $\theta > 0$, the cutoff $a_\theta(x) \in (0,1)$ is therefore well defined. It is continuous in (θ, x), strictly increasing in θ, and nonincreasing in x, and $a_\theta(x) \to 0$ as $\theta \to 0$.*

Proof. Since F_x has a positive density on $(0,1)$, $-\log[1 - F_x(c)] = \int_0^c h_x(u)\, \mathrm{d}u/u$ is finite for $c < 1$ and diverges as $c \to 1$. A positive limit of h_x at zero would make the integral infinite, and a finite limit at one would keep it bounded. As h_x is continuous and strictly increasing, it maps $(0,1)$ onto $(0,\infty)$. Its inverse inherits the joint continuity and the monotonicity of h. □

Proof of Theorem 4.2. Write $C = C_j$, $T_\theta = T_{j,\theta}$, and $V_\theta = V^j_{T_{j,\theta}}$. Fix x and put $a = a_\theta(x)$.

Part (i). The full rate burden C is continuous by Lemma A.3, the cutoff is continuous by Lemma B.1, and $0 \le C(a_\theta(r), r) \le r$, so $T_\theta \in \mathcal{T}$. Let ω be any plan from x with reports $r_1, \dots, r_N$, and let $c' = \min\{c, a\}$. Each $r_i \le x$, so $a_\theta(r_i) \ge a \ge c'$, and C is nondecreasing in its first argument. Hence

$$\sum_{i=1}^N T_\theta(r_i) + cK(\omega) \ge \sum_{i=1}^N C(c', r_i) + c'K(\omega) \ge C(c', x). \tag{51}$$

The last inequality holds because ω followed by a full rate minimizing plan from each r_i at cost c' is a plan from x. Conversely, $T_\theta \le \mathrm{id}$ gives $V_\theta(x,c) \le C(c,x)$, and truthful reporting gives $V_\theta(x,c) \le T_\theta(x) = C(a,x)$. This proves (11).

Let $c > a$ and let ω be a minimizing plan under T_θ at c. If $K(\omega) > 0$, then at a cost $c'' \in (a,c)$ the plan ω would cost less than $V_\theta(x,c) = V_\theta(x,c'')$, which is impossible. So $K(\omega) = 0$, and ω reports truthfully because every operation that changes a base requires positive effort.

Let $0 < c < a$. A full rate minimizing plan costs no more under $T_\theta \leq \mathrm{id}$ than its full rate cost $C(c, x) = V_\theta(x, c)$, so it is minimizing under T_θ. Conversely, let ω be minimizing under T_θ. Then (51) holds with equality and $c' = c$, so $C(a_\theta(r_i), r_i) = C(c, r_i)$ for each i, where $a_\theta(r_i) > c$. The concave, nondecreasing function $C(\cdot, r_i)$ is then constant on $[c, 1]$. By Lemma A.4, full rate minimizing plans from r_i at costs above c have zero effort, so they are truthful and $C(c, r_i) = r_i = T_\theta(r_i)$. The plan ω thus costs the same under the full rate schedule and is minimizing there.

Part (ii). Fix $T \in \mathcal{T}$, write $V(c) = V_T^j(x, c)$, and let $g(c) = [1 - F_x(c)][\theta - h_x(c)]$. Suppose first that $\theta > 0$. Proposition 4.1 gives

$$\mathbb{E}\big[R_T^j - (1-\theta)V_T^j \mid x\big] = \int_0^1 g(c)V'(c)\,\mathrm{d}c.$$

On $[0, a]$ the function g is continuous and strictly decreasing, as the product of the positive nonincreasing function $1 - F_x$ and the strictly decreasing function $\theta - h_x$, which is positive on $[0, a)$. Also $g(a) = 0$, and $g < 0$ on $(a, 1)$. Let ν be the measure on $(0, a]$ with $\nu((c, a]) = g(c)$. By Fubini's theorem and $V(0) = 0$,

$$\int_0^a g(c)V'(c)\,\mathrm{d}c = \int_0^a \int_{(c,a]} \nu(\mathrm{d}s)\, V'(c)\,\mathrm{d}c = \int_{(0,a]} V(s)\,\nu(\mathrm{d}s).$$

By part (i), $V_\theta(x, \cdot)$ equals $C(\cdot, x)$ on $[0, a]$ and is constant on $[a, 1]$, so the same computation applies to it. Subtracting the two expressions gives

$$\begin{aligned}\mathbb{E}\big[R_{T_\theta}^j - (1-\theta)V_{T_\theta}^j \mid x\big] - \mathbb{E}\big[R_T^j - (1-\theta)V_T^j \mid x\big] \\ = \int_{(0,a]} [C(s, x) - V(s)]\,\nu(\mathrm{d}s) + \int_a^1 [-g(c)]\,V'(c)\,\mathrm{d}c.\end{aligned} \tag{52}$$

Both terms are nonnegative because $V \leq C(\cdot, x)$ and $V' \geq 0$. They vanish if and only if $V = C(\cdot, x)$ on $[0, a]$, since ν charges every interval and $C(\cdot, x) - V$ is continuous, and $V' = 0$ almost everywhere on $(a, 1)$. This is the condition $V = V_\theta(x, \cdot)$. If $\theta = 0$, then $T_0 = 0$ yields zero revenue and burden, while $\mathbb{E}[R_T^j - V_T^j \mid x] = -\mathbb{E}[D_T^j \mid x] \leq 0$, with equality if and only if $V' = 0$ almost everywhere, that is, $V = 0$.

Part (iii). Let $\mathbb{E}R_{T_\theta}^j = G$. Integrating (12) over bases gives $\mathbb{E}R_T^j - (1-\theta)\mathbb{E}V_T^j \leq G - (1-\theta)\mathbb{E}V_{T_\theta}^j$ for every $T \in \mathcal{T}$. If T is feasible and $\theta < 1$, then $(1-\theta)[\mathbb{E}V_T^j - \mathbb{E}V_{T_\theta}^j] \geq \mathbb{E}R_T^j - G \geq 0$, so T_θ is optimal, and an optimal T raises exactly G and attains equality in (12) at μ-almost every base. If $\theta = 1$, the comparison reads $\mathbb{E}R_T^j \leq G$, so every feasible T raises exactly G and attains equality at μ-almost every base. In both cases, at such a

base $V_T^j(x,\cdot) = V_\theta(x,\cdot)$ by part (ii). Hence every feasible schedule is optimal when $\theta = 1$. Finally, $V_\theta(x,\cdot)$ is constant on $[a,1]$ and $a < 1$, so a minimizing plan under T at a cost above a has zero effort, by the argument in part (i). It is truthful, and $T(x) = C(a,x) = T_\theta(x)$. □

Proof of Corollary 4.3. Part (i). By (11) and Lemma A.4, the burden under $T_{j,\theta}$ has derivative $\partial_c C_j(c,x)$ for almost every $c < a_\theta(x)$ and zero above, so Proposition 4.1 gives (13). Since $a_\theta(x)$ is continuous in θ and the integrands are bounded by $\phi(X)[1 - F_x(u) + uf_x(u)]$, both functions are continuous on $(0,1]$ by dominated convergence, and they tend to $Q_j(0) = L_j(0) = 0$ as $\theta \to 0$. For $\theta < \eta$, the differences $Q_j(\eta) - Q_j(\theta)$ and $L_j(\eta) - L_j(\theta)$ are the integrals in (13) taken over $a_\theta(x) < u \le a_\eta(x)$, where $\theta \le h_x(u) \le \eta$. This gives (14) and both monotonicity claims. Integrating (12) at $\theta = 1$ gives $\mathbb{E}R_T^j \le Q_j(1)$ for every $T \in \mathcal{T}$, so $G_{j,\max} = Q_j(1)$. If $Q_j(\theta) = Q_j(\eta)$ for some $\theta < \eta < 1$, then $L_j(\theta) = L_j(\eta)$ by (14), so $\partial_u C_j(u,x) = 0$ for almost every u between $a_\theta(x)$ and $a_\eta(x)$, at μ-almost every x. By concavity $C_j(\cdot,x)$ is then constant on $[a_\theta(x),1]$, and Q_j is constant on $[\theta,1]$.

Part (ii). Since Q_j is continuous with $Q_j(0) = 0$, the parameter $\theta_j(G)$ exists for $0 \le G \le G_{j,\max}$, and Theorem 4.2(iii) applies to it.

Part (iii). By part (i), θ_j is continuous and strictly increasing on $[0, G_{j,\max}]$. For $0 < G < G' < G_{j,\max}$, applying (14) to $\theta_j(G) < \theta_j(G') < 1$ gives

$$\frac{G' - G}{1 - \theta_j(G)} \le L_j(\theta_j(G')) - L_j(\theta_j(G)) \le \frac{G' - G}{1 - \theta_j(G')}.$$

Hence $G \mapsto L_j(\theta_j(G))$ is differentiable on $(0, G_{j,\max})$ with the increasing derivative $1/[1 - \theta_j(G)]$. This proves (16) and the concavity of W_j^*, and it shows that minimal burden at G is $\int_0^G \mathrm{d}g/[1 - \theta_j(g)]$. □

Proof of Proposition 4.6. Fix x and $\theta > 0$, put $a = a_\theta(x)$, and let $w(u) = [1 - F_x(u)][1 - h_x(u)]$, which is continuous, nonnegative, and nonincreasing on $[0,a]$. Let ν_w be the measure on $(0,a]$ with $\nu_w((u,a]) = w(u) - w(a)$. As in the proof of Theorem 4.2(ii), Fubini's theorem gives

$$\int_0^a w(u)\, \partial_u C(u,x)\, \mathrm{d}u = w(a)C(a,x) + \int_{(0,a]} C(s,x)\, \nu_w(\mathrm{d}s). \tag{53}$$

The right side is nondecreasing in $C(\cdot,x)$, so $Q(\theta) \le \widetilde{Q}(\theta)$ by (13). Therefore $G_{\max} \le \widetilde{G}_{\max}$ and $\theta(G) \ge \widetilde{\theta}(G)$, and the cutoffs are ordered because a_θ increases with θ. By Corollary 4.3(iii), minimal burden under either technology is continuous in G, vanishes at $G = 0$, and has derivative $1/[1 - \theta(G)] \ge 1/[1 - \widetilde{\theta}(G)]$ on $(0, G_{\max})$. It is therefore weakly larger under the first technology on $[0, G_{\max}]$, which is the welfare ordering.

Every underreporting plan is feasible with splitting, so $C_S \leq C_U$ and (26) follows. A higher κ raises the effort of every plan with more than one entity, so it raises C_S. It therefore raises $G_{S,\max}$ and welfare and lowers $\theta_S(G)$ and the cutoffs. The set of costs $c < a_S(x)$ with $C_S(c,x) < C_U(c,x)$ shrinks at every base, which lowers the share in (27). For entity counts, let n_1 and n_2 be the counts of any two full rate minimizing plans at (x,c) under creation efforts $\kappa_1 < \kappa_2$. Each plan is feasible under the other creation effort, and adding the two optimality inequalities gives $c(\kappa_2 - \kappa_1)(n_2 - n_1) \leq 0$, so $n_2 \leq n_1$ when $c > 0$. By Theorem 4.2(i), a firm with a positive cost below both optimal cutoffs uses full rate minimizing plans under both creation efforts, and a firm above the cutoff for κ_2, which is the lower of the two, uses one entity under κ_2. The remaining firms have zero cost or a cost equal to a cutoff, and they have probability zero. □

Lemma B.2 (Continuity in the schedule)**.** *Let Assumption 2.1 hold and let M be as in Lemma A.3. For all $T, \widetilde{T} \in \mathcal{T}$,*

$$\|V^j_T - V^j_{\widetilde{T}}\|_\infty \leq M\|T - \widetilde{T}\|_\infty.$$

If $T_k \to T$ uniformly, then $\mathbb{E}V^j_{T_k} \to \mathbb{E}V^j_T$ and $\mathbb{E}R^j_{T_k} \to \mathbb{E}R^j_T$.

Proof. At $c > 0$, a minimizing plan under $\widetilde{T}$ has at most M reports, so its cost under T is at most $V^j_{\widetilde{T}} + M\|T - \widetilde{T}\|_\infty$. Exchanging the roles of the two schedules gives the bound, and both burdens vanish at $c = 0$. Under uniform convergence the burdens therefore converge uniformly, and so do their expectations. At each base they are concave in c, so their derivatives converge wherever the limit is differentiable (Rockafellar, 1970, Theorem 24.5), hence for almost every c. By (5), tax payments converge almost everywhere, and $0 \leq R^j_T \leq X$, so expected revenue converges by dominated convergence. □

Proof of Corollary 4.7. Continuous schedules that are linear between the thresholds of a given partition are determined by their values at the thresholds, and they are admissible exactly when each value $T(r)$ lies in $[0, r]$. They form a compact set in the uniform norm, on which expected burden and revenue are continuous by Lemma B.2. An optimum therefore exists whenever some schedule in the set raises G. If T is such an optimum, the revenue from tT is continuous in $t \in [0,1]$, zero at $t = 0$, and at least G at $t = 1$. Some t therefore raises exactly G, and $tT \leq T$ is then also optimal.

Fix $G < g < G_{j,\max}$. The linear interpolants of the continuous schedule $T^*_j(\cdot\,; g)$ on $\mathcal{P}_k$ are admissible and converge to it uniformly, so by Lemma B.2 their revenue converges to $g > G$ and their welfare to $W^*_j(g)$. Hence $\liminf_k W_{j,k}(G) \geq W^*_j(g)$. Letting $g \downarrow G$ proves the convergence, because W^*_j is concave, hence continuous, on $(0, G_{j,\max})$ and $W_{j,k}(G) \leq W^*_j(G)$. Nested partitions give nested sets of schedules. If T^*_j is nondecreasing, so are its interpolants and their downward scalings.

For the boundary, take $X = 1$, $\phi(x) = x^2$, $H = 0$, $\kappa = 1$, and independent uniform bases and costs. Here $d(x) = x^2/2 < \kappa$, so splitting offers no advantage under the full rate schedule (Corollary 4.5), and $G_{j,\max} = 1/12$. Theorem 4.2 requires every tax raising $1/12$ to equal $x^2/2$ almost everywhere. A continuous finite bracket tax would then equal this quadratic everywhere, which is impossible. □

Power-law formulas. For (4), the generalized failure rate $h_x(c) = \gamma(x)c^{\gamma(x)}/[1 - c^{\gamma(x)}]$ is strictly increasing in c. Its derivative with respect to γ has the sign of $e^z(1 - z) - 1 < 0$, where $z = -\gamma \log c > 0$. A continuous nonincreasing γ therefore satisfies Assumption 2.1(i); solving for its inverse gives (17). Equation (13) specializes to

$$\begin{aligned} Q_j(\theta) &= \mathbb{E}_x \int_0^{a_\theta(x)} [1 - (\gamma(x) + 1)u^{\gamma(x)}] \, \mathrm{d}C_j(u, x), \\ L_j(\theta) &= \mathbb{E}_x \int_0^{a_\theta(x)} (1 - u^{\gamma(x)}) \, \mathrm{d}C_j(u, x). \end{aligned} \tag{54}$$

For any admissible schedule, integration by parts in (5) gives expected manipulation cost $\gamma(x)(V_T^j(x, 1) - \mathbb{E}[V_T^j(x, c) \mid x])$. Subtracting this from mean burden proves (38); the boundary term at zero vanishes because burden is Lipschitz and starts at zero.

C Classification, organization, and tax shape

C.1 Order of operations

The accounting identity (18) gives the least effort that produces given reports when ψ is convex or concave.

Lemma C.1 (Order of operations)**.** *Let Assumption 2.1*(ii) *hold. Fix a base x and reports $r_1, \dots, r_N$ with sum $y \le x$.*

(i) *If ψ is convex, the least effort of a plan from x with these reports is*

$$H(x) + \kappa(N - 1) - \sum_{i=1}^{N} \phi(r_i) + \min\Big\{ \sum_{i=1}^{N} \psi(z_i) : z_i \ge r_i, \ \sum_{i=1}^{N} z_i = x \Big\}.$$

It is attained by splitting x into entities with bases $z_1, \dots, z_N$ and then reducing each z_i to r_i.

(ii) *If ψ is concave, the least effort is*

$$\phi(x) - \phi(y) + \kappa(N-1) + H(y) - \sum_{i=1}^{N} H(r_i).$$

It is attained by reducing x to y and then splitting y into the reports.

Proof. By (18), given the reports, the effort of a plan ω from x depends on its tree only through $\sum_e [\psi(z_e) - \psi(y_e)]$. Let $\Delta_e = z_e - y_e$ be the amount removed by reduction e. The base at any node equals the sum of the reports and of the amounts Δ_e in the subtree below it.

Part (i). Assign each reduction to one report below it, and let g_i be the total amount assigned to report i, so that $\sum_i g_i = x - y$. The reductions assigned to i lie on the path from the root to i. List them from the lowest to the highest as $e_1, e_2, \ldots$ The base after e_m is at least $r_i + \Delta_{e_1} + \cdots + \Delta_{e_{m-1}}$, and the increment of a convex function over an interval of given length is nondecreasing in the position of the interval, so

$$\psi(z_{e_m}) - \psi(y_{e_m}) \geq \psi(r_i + \Delta_{e_1} + \cdots + \Delta_{e_m}) - \psi(r_i + \Delta_{e_1} + \cdots + \Delta_{e_{m-1}}).$$

Summing over m and then over i gives $\sum_e [\psi(z_e) - \psi(y_e)] \geq \sum_i [\psi(r_i + g_i) - \psi(r_i)]$. With $z_i = r_i + g_i$, the effort of ω is therefore at least $H(x) + \kappa(N-1) - \sum_i \phi(r_i) + \sum_i \psi(z_i)$, which is the effort of splitting x into $z_1, \ldots, z_N$ and reducing each z_i to r_i.

Part (ii). List all reductions as $e_1, e_2, \ldots$ so that each comes after those above it in the tree. Then $e_1, \ldots, e_{m-1}$ lie outside the subtree below e_m, so $z_{e_m} \leq x - \Delta_{e_1} - \cdots - \Delta_{e_{m-1}}$, and by concavity

$$\psi(z_{e_m}) - \psi(y_{e_m}) \geq \psi(x - \Delta_{e_1} - \cdots - \Delta_{e_{m-1}}) - \psi(x - \Delta_{e_1} - \cdots - \Delta_{e_m}).$$

Summing over m gives $\sum_e [\psi(z_e) - \psi(y_e)] \geq \psi(x) - \psi(y)$. The effort of ω is therefore at least $H(x) - \sum_i H(r_i) + \kappa(N-1) + \psi(x) - \psi(y)$, which is the effort of reducing x to y and splitting y into the reports. □

For convex ψ, part (i) reduces the problem with splitting to a single division followed by the one-entity problem. For every schedule $T \in \mathcal{T}$,

$$V_T^S(x,c) = cH(x) + \min_{N \geq 1} \min_{z_1 + \cdots + z_N = x} \Big\{ c\kappa(N-1) + \sum_{i=1}^{N} \big[V_T^U(z_i, c) - cH(z_i) \big] \Big\}. \tag{55}$$

Indeed, the cost of a plan with reports $r_i \leq z_i$ is $\sum_i [T(r_i) - c\phi(r_i)] + c[H(x) + \kappa(N -$

$1) + \sum_i \psi(z_i)]$, and by (47) the minimum of $T(r) - c\phi(r) + c\psi(z)$ over $r \le z$ is $V_T^U(z,c) - cH(z)$. Taking all reports equal to zero, the least effort for full underreporting is $H(x) + \min_n \min_z\{\sum_i \psi(z_i) + \kappa(n-1)\}$, which equals $B(x)$ in (19) by Jensen's inequality. A minimizing count exists because a plan with n entities has effort at least $\kappa(n-1)$, while one entity needs only $\phi(x)$.

Lemma C.2 (Effort bounds)**.** *For convex ψ, B satisfies* (29)*, with ρ as in* (28)*.*

Proof. The ratio $\psi(z)/z$ is nondecreasing by convexity. It is bounded below, since splitting X into n equal entities and reducing each to zero has effort $H(X) + n\psi(X/n) + \kappa(n-1) \ge \kappa(n-1)$ by Lemma A.1. Hence $[\psi(z) + \kappa]/z$ is continuous on $(0, X]$ and tends to infinity as $z \to 0$, so its minimum ρ is attained at some z^*. For every count n,

$$n\psi(x/n) + \kappa(n-1) = n[\psi(x/n) + \kappa] - \kappa \ge \rho x - \kappa.$$

For the upper bound, which is trivial at $x = 0$, let $x > 0$ and take $n = \lceil x/z^* \rceil$. Then $x/n \le z^*$, so $\psi(x/n) \le (x/n)\psi(z^*)/z^*$, and $n - 1 < x/z^*$. Hence $n\psi(x/n) + \kappa(n-1) \le x[\psi(z^*) + \kappa]/z^* = \rho x$. □

C.2 Proof of the classification theorem

Proof of Theorem 4.4. Part (i). Let $T \in \mathcal{T}$ be subadditive, and let ω be a plan from x with reports $r_1, \ldots, r_N$ and $y = \sum_i r_i$. I show that $K(\omega) \ge \phi(x) - \phi(y) + \kappa(N-1)$. Then

$$\sum_i T(r_i) + cK(\omega) \ge T(y) + c[\phi(x) - \phi(y)] + c\kappa(N-1) \ge V_T^U(x,c) + c\kappa(N-1).$$

Hence $V_T^S \ge V_T^U$, with equality because every underreporting plan is feasible with splitting, and a plan with $N \ge 2$ is not minimizing when $c > 0$. If ψ is concave, the claim is Lemma C.1(ii), since $H(y) - \sum_i H(r_i) \ge 0$. If ϕ is concave, I show by induction from the reports upward that the effort in the subtree below a node with base z is at least $\phi(z) - \phi(\sigma) + \kappa(m-1)$, where σ is the sum and m the number of the reports below the node. This holds at a report, and it passes through a reduction because differences of ϕ add. At a split of z into $w_1, \ldots, w_n$ whose subtrees have report sums $\sigma_k \le w_k$ and report counts m_k, the split itself requires effort of at least $\kappa(n-1)$, and $(n-1) + \sum_k (m_k - 1) = m - 1$, so the effort below the node is at least $\sum_k [\phi(w_k) - \phi(\sigma_k)] + \kappa(m-1)$. Divide $[\sigma, z]$ into consecutive intervals of lengths $w_k - \sigma_k$. The kth interval begins at or above σ_k, so by concavity the increment of ϕ over it is at most $\phi(w_k) - \phi(\sigma_k)$. Summing gives the bound

$\phi(z) - \phi(\sigma) + \kappa(m-1)$. The full rate schedule is additive, so $C_S = C_U$. Then $T_{S,\theta} = T_{U,\theta}$ for every θ, and Theorem 4.2 gives the same burdens, revenue, and welfare.

Part (ii). Since $\phi = H + \psi$ is convex, $r - a\phi(r)$ is concave, and (47) gives $C_U(a,z) = \min\{z, a\phi(z)\}$. By (55), $C_S(a,x)$ is the least value of

$$J = aH(x) + a\kappa(N-1) + \sum_{i\in\mathcal{R}} [z_i - aH(z_i)] + \sum_{i\notin\mathcal{R}} a\psi(z_i)$$

over divisions $z_1 + \cdots + z_N = x$ and sets $\mathcal{R}$ of entities that report truthfully. Truthful reporting by a single entity gives $J = x$, and full underreporting gives $J = aB(x)$ at best, so it remains to show that $J \geq \min\{x, aB(x)\}$ always. Let q be the total base of the entities in $\mathcal{R}$ and $s = x - q$. If $s = 0$, superadditivity of H gives $J \geq x$. If $q = 0$, then $J \geq aB(x)$ by Jensen's inequality. Otherwise let $m \leq N-1$ be the number of entities outside $\mathcal{R}$. Superadditivity gives $J \geq q + aK$, where

$$K = H(x) - H(q) + s\zeta, \qquad \zeta = \frac{1}{s}\sum_{i\notin\mathcal{R}} [\psi(z_i) + \kappa] \geq \rho.$$

If $q + aK \geq x$ there is nothing to prove. Otherwise $aK < s$. Convexity of H and $H(0) = 0$ give $H(q)/q \leq [H(x) - H(q)]/s$, so by Lemma C.2,

$$\frac{aB(q)}{q} \leq a\left[\frac{H(q)}{q} + \rho\right] \leq a\left[\frac{H(x) - H(q)}{s} + \zeta\right] = \frac{aK}{s} < 1.$$

Splitting x into q and the entities outside $\mathcal{R}$ and then underreporting all of them fully requires effort $K + B(q)$, so $B(x) \leq K + B(q)$ and $aB(x) \leq aK + aB(q) < aK + q \leq J$. □

Under T_S^* burden is $\min\{x, \min\{c, a_S(x)\}B(x)\}$, which gives the behavior described after Theorem 4.4. For $c < c_x$ its derivative in c is $B(x)$, so by Lemma A.4 every minimizing plan has effort $B(x)$ and pays no tax. When $G > 0$, every positive report is taxed, because $a_S(r) > 0$ and $B(r) > 0$ for $r > 0$. Such a plan therefore reports zero through a division attaining $B(x)$. For $c > c_x$ burden is constant in c, so effort is zero and the firm reports truthfully.

C.3 Proof of the splitting threshold

Proof of Corollary 4.5. Part (i). For $x < x'$ put $\delta = (x' - x)/2$. Then $d(x') - d(x)$ is the sum of the increments of ψ over $[x, x+\delta]$ and $[x+\delta, x']$ minus twice its increment over $[x/2, x/2+\delta]$, which lies to the left of both. So d is nondecreasing by convexity, and it is continuous because ψ is. The perspective $t\psi(x/t)$ is convex in $t \geq 1$, so the objective

$n\psi(x/n) + \kappa(n-1)$ in (19) has nondecreasing successive differences in n, and its first difference is $\kappa - d(x)$. Its minimum is therefore attained at $n = 1$, where $B(x) = \phi(x)$, if and only if $d(x) \leq \kappa$.

Suppose $d(x) > \kappa$, so that $x > 0$ and $B(x) < \phi(x)$. For $0 < a < x/\phi(x)$, full underreporting through a division attaining $B(x)$ gives $C_S(a,x) \leq aB(x) < a\phi(x) = C_U(a,x)$. Suppose instead $d(x) \leq \kappa$ and fix $a > 0$. The function $\ell(z) = [a\phi(z) - z]_+$ is convex and vanishes at zero, so it is superadditive, and $C_U(a,z) - aH(z) = a\psi(z) - \ell(z)$. For every division $z_1 + \cdots + z_N = x$,

$$a\kappa(N-1) + \sum_i [C_U(a,z_i) - aH(z_i)] \geq a[N\psi(x/N) + \kappa(N-1)] - \ell(x) \geq a\psi(x) - \ell(x),$$

by Jensen's inequality, the superadditivity of ℓ, and $B(x) = \phi(x)$. The right side is $C_U(a,x) - aH(x)$, so $C_S(a,x) \geq C_U(a,x)$ by (55).

Part (ii). By Theorem 4.2(i), a firm with $0 < c < a_S(x)$ strictly prefers splitting under T_S^* if and only if $C_S(c,x) < C_U(c,x)$, and a firm with $c > a_S(x)$ uses one entity. If $d(x) \leq \kappa$ no cost satisfies the inequality. If $d(x) > \kappa$, every $0 < c < \min\{a_S(x), x/\phi(x)\}$ does, and these costs have positive probability because $a_S(x) > 0$ when $G > 0$. Integrating over bases gives the population statement. If H is convex, Theorem 4.4(ii) shows that the inequality holds exactly when $c < x/B(x)$, which gives (24). □

A test for splitting under an arbitrary schedule. For $T \in \mathcal{T}$ and $c > 0$, and with no curvature assumption, a single-entity plan is optimal at every base if and only if

$$V_T^U(y+z,c) - V_T^U(y,c) - V_T^U(z,c) \leq c[\kappa + H(y+z) - H(y) - H(z)] \tag{56}$$

for all $y, z \geq 0$ with $y + z \leq X$. If the inequality fails at some pair, splitting into y and z and then reducing each optimally beats every single-entity plan. If it holds, iterating it gives the corresponding inequality for a split into any number of children, and induction from the reports upward, as in the proof of Theorem 4.4(i), shows that the cost below every node with base z is at least $V_T^U(z,c)$.

C.4 Tax shape and the distribution of liabilities

Proof of Proposition 5.1. Let $B_n(x) = H(x) + n\psi(x/n) + \kappa(n-1)$. Increasing increments of convex H imply, for $0 < u < v$,

$$H(v) - H(u) \geq n[H(v/n) - H(u/n)].$$

Consequently $B_n(v) - B_n(u) \geq n[\phi(v/n) - \phi(u/n)] \geq 0$. Taking the minimum and using continuity gives monotonicity of B. Lemma C.2 proves (29); applying the increasing, one-Lipschitz map $v \mapsto \min\{x, v\}$ gives (30). With a common cost distribution, a_S is constant, so the tax is nondecreasing.

For the count comparison, put $b_n(x) = n\psi(x/n) - (n+1)\psi(x/(n+1))$. Strict convexity gives, for almost every x,

$$b_n'(x) = \psi'(x/n) - \psi'(x/(n+1)) > 0.$$

Strict perspective convexity makes $b_n(x)$ strictly decreasing in n. Since $B_{n+1} - B_n = \kappa - b_n$, an optimal count obeys $b_{n-1}(x) \geq \kappa \geq b_n(x)$, omitting the first inequality at $n = 1$. Thresholds are unique when they exist, occur in increasing order, and involve two adjacent counts. Only finitely many counts are active, by the creation-cost bound.

The cutoff is continuously differentiable by hypothesis. Differentiating $a_S B_n$ proves (32). Constant a_S and convex potentials give a nondecreasing rate within each branch. At a threshold x_n, the effort branches agree, so the $a_S' B$ terms cancel. The rate change is

$$a_S(x_n)[\psi'(x_n/(n+1)) - \psi'(x_n/n)] < 0.$$

□

Proof of Proposition 5.2. Linear H cancels from splitting costs, and

$$\frac{B(x)}{\phi(x)} = \min_{n \geq 1} \left\{ \frac{n\phi(x/n)}{\phi(x)} + \frac{\kappa(n-1)}{\phi(x)} \right\}.$$

Writing $\varepsilon_\phi(u) = u\phi'(u)/\phi(u)$, logarithmic differentiation of the first ratio gives

$$\frac{\mathrm{d}}{\mathrm{d}\log x} \log \frac{\phi(x/n)}{\phi(x)} = \varepsilon_\phi(x/n) - \varepsilon_\phi(x) \leq 0.$$

The creation term also decreases. Thus B/ϕ is nonincreasing. Before the liability cap, the tax difference has the sign of $B/\phi - \alpha_U/\alpha_S$, which can change only from positive to negative. The common cap preserves both weak inequalities, including cases with equality intervals or an empty region on one side. □

D Welfare and policy misspecification

D.1 Manipulation costs and the welfare loss

Proof of Proposition 6.1. Part (i). Fix j and x, and write $a = a_j(x)$, $\theta = \theta_j(G)$, and $K(c) = \partial_c C_j(c, x)$, which is nonnegative and nonincreasing because $C_j(\cdot, x)$ is concave. By (11) and (5), firms below the cutoff have effort $K(c)$ and firms above it have none, so

$$\mathbb{E}[D_j^* \mid x] = \int_0^a u(c)K(c)\,\mathrm{d}c, \qquad \mathbb{E}[R_j^* \mid x] = \int_0^a v(c)K(c)\,\mathrm{d}c,$$

with $u(c) = cf_x(c)$ and $v(c) = 1 - F_x(c) - cf_x(c)$. On $[0, a)$, $v > 0$ and $u/v = h_x/(1 - h_x)$ is increasing. Since K is nonincreasing,

$$\begin{aligned}&\int_0^a uK\,\mathrm{d}c \int_0^a v\,\mathrm{d}c - \int_0^a u\,\mathrm{d}c \int_0^a vK\,\mathrm{d}c \\ &\quad = \frac{1}{2}\int_0^a\int_0^a v(c)v(c')\left[\frac{u(c)}{v(c)} - \frac{u(c')}{v(c')}\right][K(c) - K(c')]\,\mathrm{d}c\,\mathrm{d}c' \le 0,\end{aligned}$$

with equality if K is constant. Since $\int_0^a v\,\mathrm{d}c = a[1 - F_x(a)]$, this is (36).

Part (ii). Both optimal schedules raise G, so $W_U^*(G) - W_S^*(G) = \mathbb{E}D_S^* - \mathbb{E}D_U^*$, which is nonnegative by Proposition 4.6 and at most $\mathbb{E}D_S^*$. Integrating (36) over bases gives $\mathbb{E}D_S^* \le G \sup_x \lambda_x(a_S(x))$.

Part (iii). Under the power law, $\int_0^a cf_x(c)\,\mathrm{d}c = \gamma a^{\gamma+1}/(\gamma+1)$ with $\gamma = \gamma(x)$, and $a^\gamma/(1 - a^\gamma) = \theta/\gamma$ at $a = a_\theta(x)$ by (17). Hence $\lambda_x(a_\theta(x)) = \theta/(\gamma(x) + 1)$, and part (ii) gives (37).

For sharpness, fix $\gamma > 0$, $0 < \theta \le 1$, and $a = [\theta/(\gamma + \theta)]^{1/\gamma}$. Let $\phi(r) = r^2/4$, $H = 0$, $\kappa = 1$, and place all firms at an even integer base X. The arithmetic–geometric mean inequality, with $n = X/2$, gives $B(X) = X - 1$. Also $B(r) \le r$ by (29) with $\rho = 1$, so the splitting tax at cutoff $a < 1$ has a slack cap. Set

$$G_X = a(1 - a^\gamma)(X - 1).$$

Its optimal parameter with splitting is θ. The full rate burden $C_S(c, X) = c(X - 1)$ is linear below the cutoff, so part (i) holds with equality and $\mathbb{E}D_S^* = \theta G_X/(\gamma + 1)$. Without splitting, let α_U be the smaller root of

$$\alpha_U(1 - \alpha_U^\gamma)X^2/4 = G_X.$$

It exists for large X, lies on the increasing-revenue branch, and satisfies $\alpha_U \sim 4a(1 - a^\gamma)/X$.

Since $\alpha_U X/4 \to a(1-a^\gamma) < 1$, the no-splitting cap is slack throughout $[0, X]$ for large X, and $C_U(c, X) = cX^2/4$ is linear below the cutoff. Part (i) again holds with equality, at the parameter $\theta_U(G_X) = \gamma\alpha_U^\gamma/(1-\alpha_U^\gamma) \to 0$. Thus $\mathbb{E}D_U^*/G_X \to 0$ and $(W_U^* - W_S^*)/G_X \to \theta/(\gamma+1)$ as $X \to \infty$. □

D.2 Revenue from the original schedule

Proof of Proposition 6.2. Fix x and write $a = a_U(x)$ and $\theta = \theta_U(G)$, so that $T_U^* = T_{U,\theta}$ and $\theta > 0$.

Part (i). Let $c \le a$ and let $\omega \in \Omega_S(x)$ have reports $r_1, \dots, r_N$. Since $C_U \ge C_S$ and $a_U(r_i) \ge a \ge c$, the argument for (51) gives

$$\sum_i T_U^*(r_i) + cK(\omega) \ge \sum_i C_S(c, r_i) + cK(\omega) \ge C_S(c, x).$$

The reverse inequality follows from $T_U^* \le \mathrm{id}$. For $c \ge a$, the realized burden is nondecreasing in c, so it is at least $P^M(x, a) = C_S(a, x)$, and truthful reporting bounds it by $T_U^*(x) = C_U(a, x)$.

Part (ii). Let ν be the measure in the proof of Theorem 4.2(ii). That proof shows that $\mathbb{E}[R^j_{T_{j,\theta}} - (1-\theta)V^j_{T_{j,\theta}} \mid x] = \int_{(0,a]} C_j(s, x)\, \nu(\mathrm{d}s)$ for either technology, and this integral is weakly smaller for $j = S$ because $C_S \le C_U$. The schedule T_U^* is admissible, so Theorem 4.2(ii) for the technology S gives

$$\begin{aligned}\mathbb{E}[R^M - (1-\theta)P^M \mid x] &\le \int_{(0,a]} C_S(s, x)\, \nu(\mathrm{d}s) \\ &\le \int_{(0,a]} C_U(s, x)\, \nu(\mathrm{d}s) = \mathbb{E}[R^U - (1-\theta)P^U \mid x].\end{aligned}$$

Rearranging gives (40), where $m_x \ge 0$ because every underreporting plan remains feasible. Integrating over bases gives (41).

Part (iii). Under the conditional power law, subtracting (38) for the two burdens gives

$$\mathbb{E}[R^U - R^M \mid x] = (\gamma(x) + 1)m_x - \gamma(x)e_x.$$

Integrate to obtain (42), which is nonnegative by part (ii). Since burden equals taxes plus manipulation costs, subtracting mean burden relief yields (43). □

D.3 Severe fiscal failure

Proof of Proposition 6.3. With uniform costs the generalized failure rate is $h(c) = c/(1-c)$, so the cutoff $b/X \le 1/2$ corresponds to a parameter in $(0,1]$. By Theorem 4.4(ii), $C_U(b/X, r) = \min\{r, br^2/X\} = br^2/X$ on $[0, X]$, so (44) is the no-splitting optimum for the revenue it raises, $(1-b/X)(b/X)X^2 = b(X-b)$. With splitting, $B(X) = \min_n\{X^2/n + n - 1\} = 2X - 1$, attained at $n = X$, and $C_S(a, X) = \min\{X, a(2X-1)\}$ has a slack cap for $a \le 1/2$. Maximum revenue is therefore $\max_{a \le 1/2} a(1-a)(2X-1) = (2X-1)/4$, which exceeds G_X by $(1/2-b)(X-1/2-b) > 0$.

Now let firms split under $T = T_U^*$. Since $\psi = \phi$ is convex, (55) applies with $V_T^U(z, c) = \min\{b/X, c\}z^2$ from (47), and equal entities minimize $\sum_i z_i^2$. The realized burden at base X is therefore

$$P(c) = \min_{n \ge 1}\Big\{c(n-1) + \min\{b/X, c\}\frac{X^2}{n}\Big\}.$$

For $c < b/X$ firms underreport fully and $P(c) = c(2X-1) < 2b$. For $c \ge b/X$ they report the whole base and $P(c)$ is the minimum of (46). There $bX/n + cn \ge 2\sqrt{bXc}$ for every n, while $n = \lceil \sqrt{bX/c} \rceil$ gives $bX/n \le \sqrt{bXc}$ and $c(n-1) < \sqrt{bXc}$. Hence

$$2\sqrt{bXc} - c \le P(c) \le 2\sqrt{bXc} \quad \text{for } c \ge b/X, \qquad \big|P(c) - 2\sqrt{bXc}\big| < 2b \quad \text{for } c < b/X.$$

Consequently $-1 \le P(1) - 2\sqrt{bX} \le 0$ and $-1/2 - 2b^2/X \le \int_0^1 P(c)\,\mathrm{d}c - \frac{4}{3}\sqrt{bX} \le 2b^2/X$. With uniform costs, (38) gives $Q_X^M = 2\int_0^1 P(c)\,\mathrm{d}c - P(1)$ and $D_X^M = P(1) - \int_0^1 P(c)\,\mathrm{d}c$. Both therefore differ from $\frac{2}{3}\sqrt{bX}$ by at most $1 + 4b^2/X < 2$, which is (45). Since $G_X = b(X-b)$ grows in proportion to X, the ratio Q_X^M/G_X is of order $X^{-1/2}$. □

D.4 Exact calculations for the examples and figures

In the running example, x takes values $1, 2, 3, 4$ with equal probability, c is uniform, and $\phi(x) = x^2$, $H = 0$, $\kappa = 1$. The branch $B_n(x) = x^2/n + n - 1$ is active between adjacent thresholds $\sqrt{(n-1)n}$ and $\sqrt{n(n+1)}$. At integer x, $B(x) = 2x - 1$, so $\mathbb{E}B = 4$ and $\mathbb{E}\phi = 15/2$. The cutoffs

$$\alpha_S = 1/4, \qquad \alpha_U = (1 - \sqrt{3/5})/2$$

therefore both raise $G = 3/4$, using $Q_j = \alpha_j(1-\alpha_j)\mathbb{E}B_j$, with $B_U = \phi$ and $B_S = B$. Their mean manipulation costs are $\alpha_j^2\mathbb{E}B_j/2$. Both liability caps are slack on $(0, 4]$.

Under the original tax $T_U^*(r) = \alpha_U r^2$, firms with costs below α_U underreport the entire

base. Above it, count n minimizes $\alpha_U x^2/n + c(n-1)$ and is optimal on

$$I_n(x) = \left[\frac{\alpha_U x^2}{n(n+1)}, \frac{\alpha_U x^2}{(n-1)n}\right] \cap [\alpha_U, 1],$$

where the upper endpoint is infinity for $n = 1$. Conditional revenue and manipulation costs are obtained by integrating $\alpha_U x^2/n$ and $c(n-1)$ over these intervals, adding full-underreporting costs below α_U. The four-base population average gives the revenue and costs described in Section 6. The largest one-entity threshold is $8\alpha_U < 1$, so all highest-cost firms comply and the endpoint correction vanishes. The fiscal-failure figure uses the same exact cost intervals with $\alpha_U = b/X$ and a common integer base X. All marginal-rate curves are computed from their analytic branch derivatives.

E Evidence from a registration threshold

E.1 Responses to a threshold

Proof of Proposition 7.1. Let ω be a plan from x with reports $r_1, \ldots, r_N$. With $H = 0$, (18) gives $K(\omega) = \kappa(N-1) + \sum_e [\phi(z_e) - \phi(y_e)] \geq \kappa(N-1)$, and the inequality is strict if some reduction lowers a base, because ϕ is strictly increasing. Suppose first that $N \geq 2$. The cost of ω is then at least $c\kappa$. It equals $c\kappa$ only if $N = 2$, no reduction lowers a base, and no tax is paid, in which case the two reports are the bases formed by one split, are at most b, and sum to x. Splitting x into two entities with base $x/2 \leq b$ that report their bases is such a plan. Suppose next that $N = 1$. The plan then contains no split, and by path independence its effort is $\phi(x) - \phi(r)$, where r is its report. If $r \leq b$, its cost is $c[\phi(x) - \phi(r)] \geq c[\phi(x) - \phi(b)]$, with equality only if $r = b$. If $r \in (b, x]$, its cost is $g(r) = \tau r + c[\phi(x) - \phi(r)]$. Because ϕ is convex, g is concave on $[b, x]$, so $g(r) \geq \min\{g(b), g(x)\}$, where $g(x) = \tau x$ and $g(b) = \tau b + c[\phi(x) - \phi(b)] > c[\phi(x) - \phi(b)]$. Every plan therefore costs at least the smallest of the three terms, and each term is the cost of some plan, which proves the formula.

If $c\kappa$ is the unique minimum, no plan with one report attains it, and the plans with $N \geq 2$ that attain it have two reports that are at most b and sum to x. If $c[\phi(x) - \phi(b)]$ is the unique minimum, a plan that attains it has one report r, and $r > b$ is impossible because $g(r) \geq \min\{g(b), g(x)\}$ exceeds this term. Hence $r = b$. If τx is the unique minimum, a plan that attains it has one report $r > b$, and $g(b) > \tau b + \tau x > \tau x$. For $r \in (b, x)$, concavity gives $g(r) \geq \lambda g(b) + (1-\lambda) g(x) > \tau x$ with $\lambda = (x - r)/(x - b) > 0$. Hence $r = x$. □

The next result covers bases slightly above twice the threshold, for which Section 7

states that a firm that manipulates has two entities at the threshold.

Lemma E.1 (Bases slightly above twice the threshold)**.** *Let the assumptions of Proposition 7.1 hold, and suppose that $X > 2b$ and $\phi(2b) - \phi(b) > \kappa$. There is $\delta > 0$ such that, for every base $x \in (2b, 2b + \delta)$ and every cost parameter $c \in (0, 1]$ with $c\{\kappa + 2[\phi(x/2) - \phi(b)]\} < \tau x$, every minimizing plan under T_b has two entities that both report b.*

Proof. Write $m(x) = \kappa + 2[\phi(x/2) - \phi(b)]$. By continuity there is $\delta \in (0, X - 2b)$ such that $2[\phi(x/2) - \phi(b)] < \min\{\kappa, \tau b\}$ and $m(x) < \phi(x) - \phi(b)$ for all $x \in (2b, 2b + \delta)$. Fix such a base and a cost parameter as in the statement. Splitting x into two entities of base $x/2$ that each underreport to b costs $cm(x)$. As in the proof of Proposition 7.1, a plan with one report costs at least $\min\{\tau x, c[\phi(x) - \phi(b)]\} > cm(x)$, and a plan with three or more reports costs at least $2c\kappa > cm(x)$. A plan with two reports of which one exceeds b pays more than τb in tax and requires effort of at least κ, so it costs more than $c\kappa + \tau b \geq c\kappa + c\tau b > cm(x)$.

It remains to consider plans with two reports $r_1, r_2 \leq b$. Such a plan contains one split, of some base $z \leq x$ into w_1 and w_2, with reductions before and after it, so its effort is $\kappa + \phi(x) - \phi(z) + \sum_i [\phi(w_i) - \phi(r_i)]$. Because ϕ is strictly increasing, $\phi(r_i) \leq \phi(b)$ with equality only if $r_i = b$. Because ϕ is convex, $\phi(w_1) + \phi(w_2) \geq 2\phi(z/2)$ and $\phi(x) - \phi(z) \geq 2[\phi(x/2) - \phi(z/2)]$, since increments of a convex function over intervals of equal length do not decrease as the interval moves to the right. The effort is therefore at least $m(x)$, with equality only if both reports equal b. □

The division of a firm that splits and reports its whole base is not determined when $H = 0$. Suppose instead that H is strictly convex. A plan that splits $x \in (b, 2b]$ once into bases $w_1, w_2 \leq b$ and reports them requires effort $\kappa + H(x) - H(w_1) - H(w_2)$. The sum $H(w_1) + H(x - w_1)$ is strictly convex in w_1 on $[x - b, b]$ and symmetric around $x/2$, so it is largest, and the effort is smallest, exactly when one base is b and the other is $x - b$.

E.2 Data

I take ownership histories from the Register of Legal Entities, which includes the commercial register, in the complete extract published by Slovensko.Digital.[6] I take annual accounts from the Register of Financial Statements through its public interface.[7] VAT registrations come from the list published by the Financial Administration, the Slovak tax administration, in the Slovensko.Digital extract, which records when a company enters and leaves the list from June 2016 onward. I count a company as registered at a given date

[6] https://ekosystem.slovensko.digital/otvorene-data
[7] https://www.registeruz.sk/cruz-public/home/api

if the list contains it on that date as a taxable person registered under Section 4 of the VAT Act. As a check on this measure, consider the companies in the panel described below that had sales above EUR 100,000 in a year and are outside the industries with exempt services. Of these, 93 percent are classified as registered at the end of 2016 and 96 to 97 percent at the end of each year from 2017 to 2022.

Owners are natural persons recorded as shareholders of limited liability companies. I treat shareholder records with the same full name and address as one person. I join two records with the same name and different addresses only if they are consecutive entries in the shareholder history of one company and the second begins on the day after the first ends. The registers contain no personal identifier, so an owner who is recorded under different addresses in different companies is not linked, and two persons who share a name and an address are not distinguished.

The panel of owners starts from the 108,196 owners who were the sole owner of exactly one active limited liability company on 31 December 2016. An owner is at risk of forming a second company in a year if, at the start of that year, the owner is still the sole owner of the first company and has held no other limited liability company since 2016. The event is the incorporation of a company of which the owner is the sole shareholder at incorporation. The sample contains all 3,165 owners who became a shareholder of a newly incorporated company between 2017 and 2023 while owning the first, and 2,887 of the remaining 105,031 owners drawn at random, each with weight 105,031/2,887. It has 25,811 owner-years at risk and 1,834 events. The sales of the first company in the previous year are available for 21,058 of these owner-years and 1,656 of the events. The accounts cover every company of the sampled owners from 2016 to 2023.

The cross-section starts from all owners who were the sole owner of at least one limited liability company on 31 December 2019 and is composed as described in Section 7. For each company I retrieved the accounts for the financial year 2020, which report 2019 in a comparative column, and the accounts for 2019 or 2021 where these were missing. I use the accounts of a company for a year if they cover January to December and contain an income statement. They are available for 28,376 companies for 2019 and for 28,409 companies for 2020. For 9,930 of the 40,874 companies the register contains no accounts for these years.

Sales are the sum of the income statement items for sales of goods and for sales of own products and services. Revenue, which I compare with the ceiling of the lower income tax rate, is the sum of operating revenue and financial revenue. I compute the ratio of tax to profit for companies with a profit before tax above EUR 2,000. The industry of a company is the two-digit division of its NACE code in the Register of Legal Entities, which records no code for 6 percent of the companies with accounts for 2019. Comparisons of industries

use the companies with a code. Industries with exempt services are divisions 64 to 66 (finance and insurance), 68 (real estate), 85 (education), 86 to 88 (health and social work), and 92 (gambling). Of the 610 unregistered companies with sales between one and two times the threshold in 2019, 43 percent belong to these industries and a further 16 percent registered during 2020. The remainder can have other sales that do not count toward the threshold, or can have failed to register.

Confidence intervals are percentile intervals from 2,000 bootstrap samples of owners, drawn separately among the owners who became a shareholder of a new company and the comparison owners. Random pairing permutes the second company of each owner across owners 5,000 times. For owners of three companies, I recombine the smallest, the middle, and the largest companies across owners at random, also 5,000 times.

E.3 Additional results

The difference in the probability of forming a second company is similar when industries with exempt services are excluded and when the second company can have other shareholders (Table 2). It is larger in 2017 and 2018 than from 2019 to 2023. The difference is also present within age groups. Take the owners whose first company had sales within 20 percent below the threshold. When that company is at most five years old, the annual probability of forming a second company is 1.7 percent if it is unregistered and 0.7 percent if it is registered. The two probabilities are 0.8 and 0.5 percent when it is six to ten years old, and 0.6 and 0.1 percent when it is older. The number of owners with both companies near the threshold is 2.6 times the number under random pairing in 2019, with or without industries with exempt services, and 2.2 times in 2020.

Table 2: The main comparisons for other samples and definitions

A. Annual probability of forming a second company, percent				
	Unregistered	Registered	Difference	95 percent interval
Sales of the first company within 20 percent below the threshold				
All owners, 2017 to 2023	1.06	0.39	0.67	[0.42, 0.95]
Without exempt industries	1.08	0.38	0.70	[0.44, 1.01]
Other shareholders allowed	1.39	0.59	0.81	[0.49, 1.17]
2017 and 2018	1.54	0.36	1.18	[0.75, 1.66]
2019 to 2023	0.77	0.40	0.37	[0.11, 0.67]
Sales of the first company between EUR 45,000 and the threshold				
All owners, 2017 to 2023	1.30	0.47	0.83	[0.45, 1.29]

B. Owners with both companies within 20 percent below the threshold			
	Owners	Observed	Random pairing, mean (s.d.)
2019	4,602	77	30.0 (5.0)
2019, without exempt industries	3,679	67	25.4 (4.5)
2020	4,493	57	25.7 (4.6)

Notes. Panel A refers to sole owners of one company, by the sales of that company in the previous year and its VAT registration at the end of that year. In the third row the second company can have other shareholders. Panel B counts sole owners of two companies both of which have sales between 80 and 100 percent of the threshold. Exempt industries are the industries with services exempt from VAT that are listed in the text.